TWIN PARADOX

The "Twin Paradox" asserts that time dilatation is a physical reality. But how can someone who is returning from a space journey find his twin brother -who has stayed on earth- being older than himself, though Einstein's Special Relativity is based on the premise that no reference system is privileged? That is to say: Why doesn't the other twin brother, too, have the "right" of being the younger one? This article tries to provide an answer.

In Special Relativity, a way is found to harmonize two fundamental principles, which seem to be contradict each other at first sight. The first one is the law of constant propagation of light. In its original form, it claims that any observer watches light beams propagating at a velocity c, no matter if the source of light is moving or is at rest, provided it can be excluded that the medium itself (through which the waves are propagating) is moving toward him or away from him. The second principle is the relativity principle. It claims that any observer who is not subject to acceleration can consider himself as being at rest. To put it differently: it claims that there are no such things like absolute rest or absolute motion.

It shall be demonstrated why the two principles appear to be incompatible. It is, first of all, easy to imagine light propagating at constant velocity for an observer at rest, as long as a supposed medium, too, is at rest with respect to that observer. Waves created by someone who, sitting on a boat, is throwing pebble stones into the water of a calm pond, propagate at constant speed for an observer on the shore. For this observer, the speed of the waves is the same whether or not the boat is moving when the stones are being dropped. It is, however, hard to imagine how a lightbeam emitted by a first observer can have the same velocity c for that first and also for a second observer, even though the second observer is running after the head of that lightbeam trying to catch up with it. Provided that the velocity of the lightbeam is c for the first observer, intuition would tell us that the velocity of the lightbeam would be less than c for that second observer. We would be inclined to say that either the first or the second observer (who is trying to catch up with the head of the lightbeam) cannot be at rest relative to the medium through which light is moving. He can therefore not expect that the law of constant propagation of light (which applies to observers at rest only) can be applied with respect to him. **Different from the water waves, however, there is no medium for the propagation of light. With no medium existing, a velocity cannot be**

acsribed to a medium. To put it differently: There is no medium that could move toward or away from an observer. By nevertheless maintaining that the second observer does not have the right of applying the law of constant propagation of light also to **his** perspective, we would assert that the first observer is privileged somehow. This is equivalent to saying that there are such things like absolute motion and absolute rest. We would refuse the second observer his "right" to consider himself as being at rest rather than in motion. Yet for our intuition, giving up the relativity principle (according to which there is no absolute rest) seems to be the only way to save the law of constant propagation of light, after we have discovered that there is no medium for the transport of light to which a velocity could be ascribed. We would thus ascribe a role to the notion of absolute rest that was previously played by the (supposed) medium.

Fortunately, the two principles are only incompatible for our intuition. They are not incompatible in a logical sense. The easiest way to show this is by means of a spacetime diagram. In their most simple form, spacetime diagrams have an abscissa representing a spatial direction and an ordinate representing time. Imagine you live on the embankment of a railroad track. Your spatial position could then be determined by one number alone, that is by the milestone of the railroad track next to your house. If you stay at home all life long, your life could be represented by a vertical graph on a spacetime diagram. If you travel along the railroad track from time to time, your "life"-graph will be curved. Instead of "life"-graph, we will use the term "worldline".

What shape can we expect for the worldline of a short lightbeam transmitted from its source at milestone 0 at time 0? It depends on how we choose the measuring units for each axis of the diagram. Choosing lightyears (=c times one year) for the spatial x-axis and years for the temporal t-axis, the worldline of the lightbeam will bisect the angle formed by the two axes, if and only if the speed of that lightbeam is c (see fig. 1).

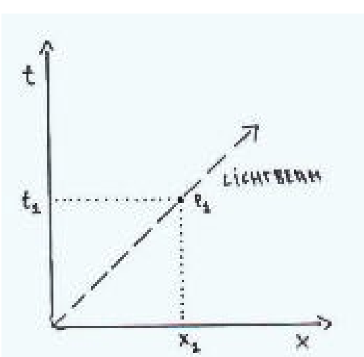

Fig. 1

Rather than using axes that cross at a right angle, one can apply axes that cross at an angle more narrow (see fig. 2). If we wish to determine the time (t-) coordinate of P_1, we have to draw a parallel to the x-axis and make this parallel run through point P_1. At point t_1 this parallel meets the t-axis; and thereby our task is completed. This procedure may appear strange at first glance. Yet when we intend to determine the t-coordinate of the point P_1 in fig. 1, we do not behave differently. We draw a parallel to the x-axis, which, in that coordinate system, is at right angle with the x-axis. Logic does not confine diagrams to those shown in fig. 1. As long as we use the same measuring units for the axes as we did in fig. 1, the worldline of the short lightbeam will bisect the angle formed by the two axes also in fig. 2.

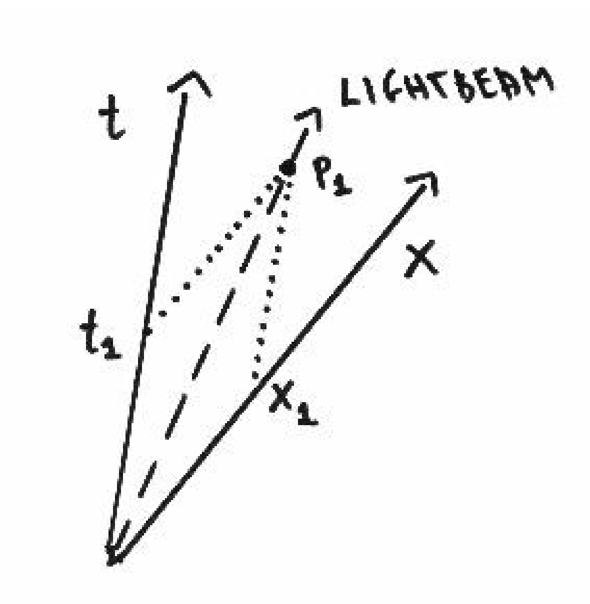

Fig. 2

When combining fig. 1 with fig. 2, we get two integrated coordinate-systems as shown in fig. 3:

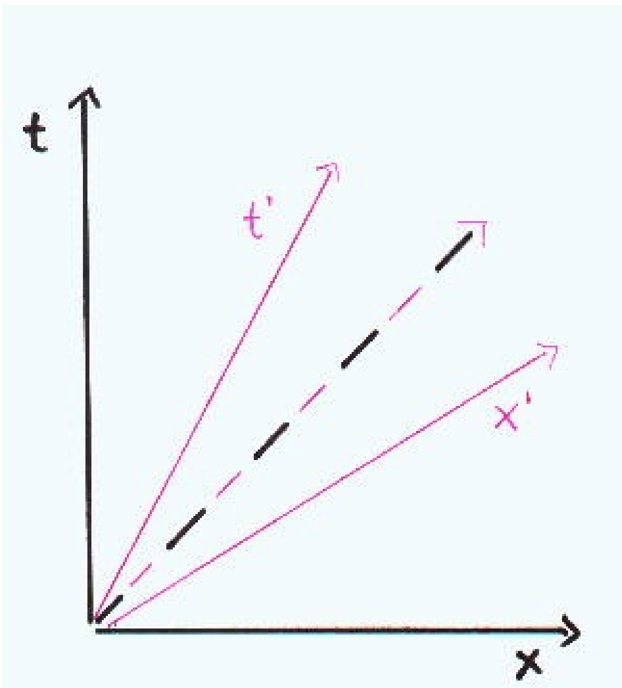

FIG. 3

Imagine that the worldline of a first observer is identical with the vertical t- axis in figure 3. The worldline of a second observer shall be identical with the inclined t'-axis. Then, viewed from the perspective of the first observer, the second observer is moving away at constant speed (and vice versa). Right now fig. 3 reveals an important feature: it tells us (contrary to our intuition) that both observers, though not at rest with regard to each other, watch the same short lightbeam moving with the same velocity c (relative to each observer)!

The "price" paid for that sameness of velocity is the difference in "clock-speed": Suppose one inch on the t- and also on the t'-axis in figure 3 would equal 1 year each. For the first observer, whose worldline is identical with the vertical t-axis, HORIZONTAL lines mark simultaneous events (see fig. 4 which shows a section of fig. 3).

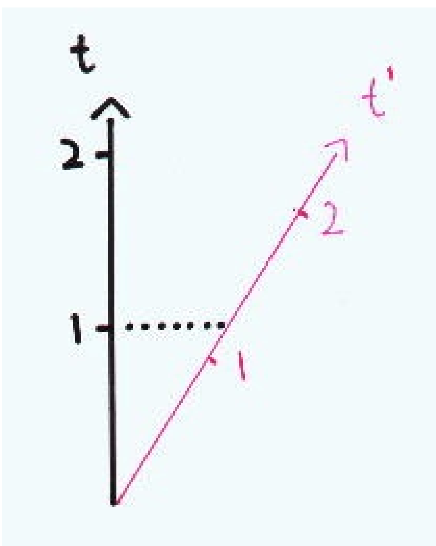

Fig. 4

Hence, after the elapsing of the first year (both clocks were set zero when the two observers split from each other at the origin of coordinates), he would have to come to the result that more than one year (1.2 years) must have elapsed for the second observer, whose inclined worldline is identical with the t'-axis (see fig. 4 again). To put it differently: the first observer would watch a "compression" of time going on in the reference system of the second observer.

On the other hand, that second observer would judge that less than one year has elapsed for the first observer while one year has elapsed for his own clock: his simultanity line is inclined (red dotted line in fig. 5), as it represents a parallel to the x'-axis (see fig. 3 again). In other words: the second observer would watch a dilatation of time going on in the reference system of the first observer (see fig. 5 below).

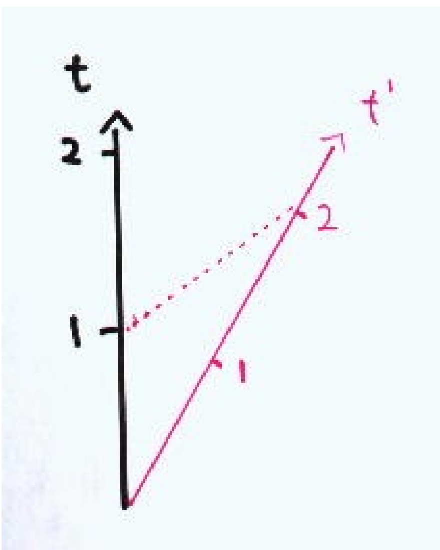

Fig. 5

This asymmetry, however, would contradict the relativity principle. Both observers have the same right to consider themselves at rest, so there is no reason why one observer should watch a compression of time (taking place in the reference system of his counterpart) while the other observer is watching a dilatation (taking place in the reference system of his counterpart). Therefore one inch cannot represent one year on both the t- and the t'-axis.

How could we modify our diagram in order to guarantee equal rights for each of the two observers? We simply have to change the scale on the axes: an interval of one year shall no longer be represented by one inch in BOTH reference systems. As soon as we stretch the one year mark on the t'-axis to a certain degree (leaving the one year mark on the t-axis in place), we reach the following two results (see fig. 6 below):

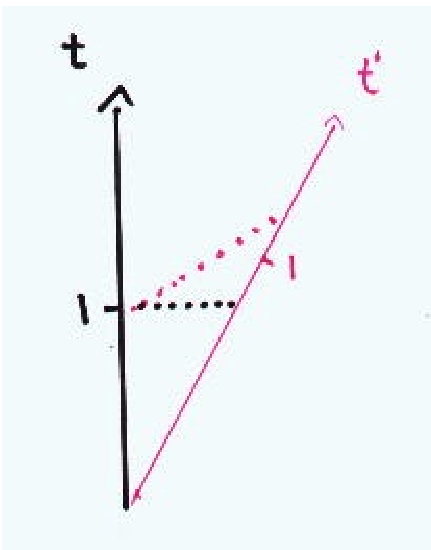

FIG. 6

1. Each observer watches time being dilatated in the other system.

2. The factor of dilatation (which the other reference system is subject to) is the same for both observers.

But now the twin paradox appears to be farther away from being resolved than ever before. For it claims (in an apparent contradiction to the result just found) that one twin, after having returned to earth from a space trip, will be younger than the one who stayed at home. According to the twin paradox, there is no way to reverse that relationship. To put it differently: there is no way for the one who stayed at home to consider himself younger than the returning traveler. To put it in a third manner: the twin paradox seems to refuse equal rights for both of the two observers.

Yet the first impression is false. Take a look at fig. 7 below:

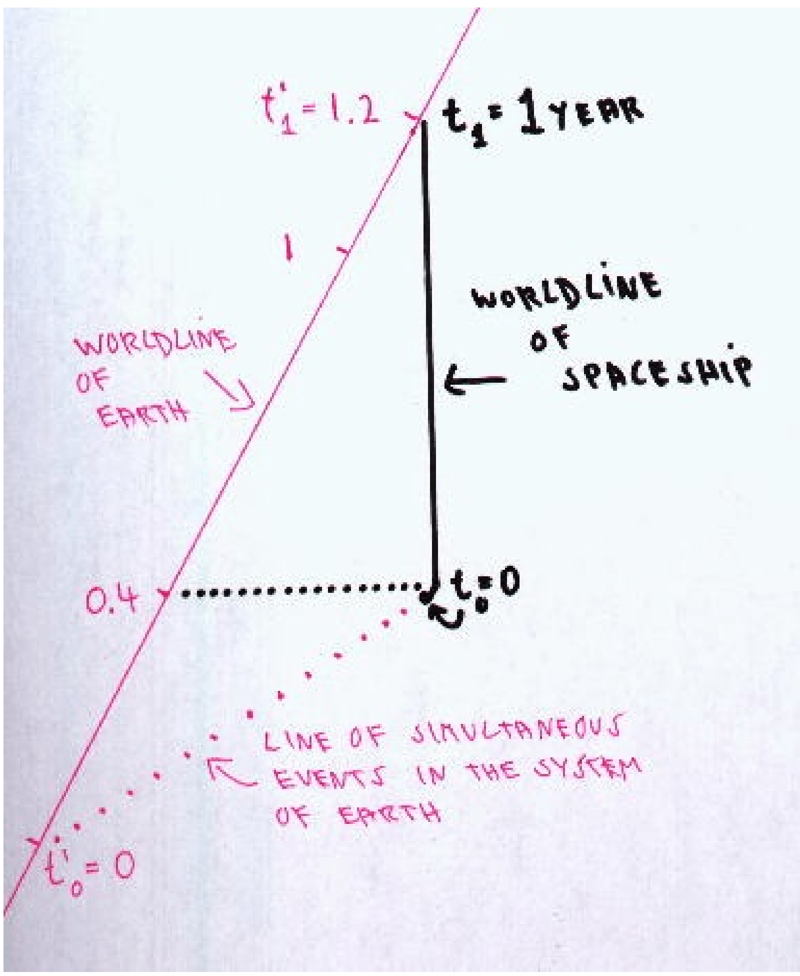

Fig. 7

At t_0, we see a spaceship starting out from a distant planet for its return to earth. That planet (the worldline of which is not shown in figure 7) is at rest relative to earth. The worldline of the spaceship has been chosen to be vertical; hence the worldline of planet earth is inclined (the wordline of the distant planet would be a parallel to the worldline of planet earth). Clocks on that distant planet shall be in conformity with those on earth and show time zero when the spaceship is about to take off from the surface of that planet. Fig. 7 pictures the return trip from the moment the spaceship begins its short interval of acceleration (represented by a short curve) until its arrival on earth. After the short period of acceleration (over which only a small distance was covered), clocks on the distant planet are practically still at zero. So is the clock on the spaceship. That is to say: when the crew and the inhabitants of that planet compare the settings of their clocks over the wireless, they still find conformity. But now comes the surprise. When the crew want to figure out what time it is right now (= at spaceship's time zero) on earth, they have to come to the result that clocks on earth have made a big jump forward as an effect of the short acceleration which the spaceship underwent!!! To put it differently: for the crew, all of a sudden the clocks on distant earth are 0.4 years ahead of time.

When the spaceship is approaching planet earth, the clocks on board are having the tendency of catching up with the clocks on earth. That means: the time gap between the clocks on board and those on earth is constantly becoming smaller. This is quite "natural": time on earth is dilated from the perspective of the crew on the spaceship.

When destination earth is reached, the clocks on the spaceship have not managed to catch up with the ones on earth completely; a certain time gap is still existent. This is why people on earth judge that time was dilated on the spaceship. The crew, however, is sure that time was dilated on earth, since -during the trip- the clocks on board managed to diminish their temporal lag behind the ones on earth. As already pointed out, the clocks on earth had made a big jump forward (viewed from the spaceship) when the spaceship was accelerated by its engines.

This is the solution of the twin paradox.

One might ask oneself whether that time dilatation is "real" or whether it is a mere fiction necessary to combine the two principles mentioned above. Richard P. Feynman has given an answer (Lectures on Physics, Vol. 1, chapter 15-4):

"The biologists and medical men sometimes say it is not quite certain that the time it takes for a cancer to develop will be longer in a space ship, but from the viewpoint of a modern physicist it is nearly certain; otherwise one could use the rate of cancer development to determine the speed of the ship!"

In other words: if there were just one single physical process on board of the ship that is not subject to dilatation, the notion of absolute motion would become a physical reality, and the relativity principle would have to be given up.

Please send comments to

Andreas Trupp
atrupp@aol.com

For a pdf-version of this article (with text and figures all squeezed into one single file) **click here !**