PHYSICS ESSAYS **33**, 4 (2020)

Traversing a black-and-white hole in free fall and rise

Andreas Trupp[a)]
andreas@andreastrupp.com

(Received 2 July 2020; accepted 24 October 2020; published online 19 November 2020)

Abstract: It is shown that a traverse of a Black-and-White Hole (through a shaft in the interior of the central, spherical body) in free radial fall and rise is described by the Schwarzschild metric without any ambiguity. In other words, all Black Holes can also be White Holes. The relativity principle, according to which both the freely falling/rising observer Alice and a second observer Bob (sitting outside of the gravity field) have to measure the same temporal interval for the complete trip, is observed $[(\Delta t)/(\Delta\tau)=1]$. In the interior of the Schwarzschild radius, Alice's time τ is reversed. Kruskal charts do not present an obstacle to this result, since quadrant II can be used for ingoing traffic only, but not for outgoing traffic. © *2020 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-33.4.460]

Résumé: Il est montré qu'une traversée d'un trou noir et blanc (à travers un puit à l'intérieur du corps sphérique central) en chute et montée radiales libres est décrite par la métrique Schwarzschild sans aucune ambiguïté. En d'autres termes, tous les trous noirs sont également des trous blancs. Le principe de relativité, selon lequel l'observateur Alice qui tombe ou monte librement et un deuxième observateur Bob (situé à l'extérieur du champ de gravité) doivent mesurer le même intervalle temporel pour le voyage complet, est observé $[(\Delta t)/(\Delta\tau)=1]$. À l'intérieur du rayon Schwarzschild, le temps τ d'Alice est inversé. Les diagrammes Kruskal ne présentent pas d'obstacle à ce résultat, car le quadrant II ne peut être utilisé que pour le trafic entrant, mais pas pour le trafic sortant.

Key words: General Relativity; Black Holes; Relativity Principle; Time Reversal; Kruskal Charts; Free Fall.

I. INTRODUCTION

Black Holes are considered as objects from which no objects or signals can escape. The only exception that is conceded to exist is to be a mere theoretical possibility: Since all processes in nature are reversible in time, a Black Hole is assumed do the opposite of what it is usually doing (for a short period of time) if one waits long enough: it must then "spit out" objects and signals while it does not allow any object or signal to enter. Such an object has been labeled "White Hole," and it is considered as a phenomenon that is very improbable to occur.

The confidence in the blackness of Black Holes has been gained by so called Kruskal-charts. Kruskal-charts are geometrical representations of the result of a coordinate exchange. The t- and r-coordinates of the Schwarzschild metric are substituted by a timelike T- and a spacelike X-coordinate. The so-generated chart is interpretated in a way that (within the Schwarzschild radius of the massive, spherical body) an outgoing light pulse or any other outgoing object is doomed to end up at the center of the spherical body.

It will be shown that this interpretation of Kruskal-charts is wrong, namely, at odds with the Schwarzschild metric.

a)andreas@andreastrupp.com

II. DERIVING THE EQUATION OF FREE FALL/RISE WITH "COORDINATE TIME" *t* OF A DISTANT OBSERVER AS THE DEPENDENT VARIABLE

According to Newtonian physics, the potential energy W_{pot} per unit mass of a test body that finds itself in free radial fall/rise outside of a massive spherical body at a distance $r > r_0$ from the center is equal to (r_0 is the radius of the spherical body, measured as circumference divided by 2π)

$$\begin{aligned} -W_{\text{pot}}(r) &= \int_r^\infty \frac{GM}{r^2}\,dr \\ &= GM\left[-\frac{1}{r}\right]_r^\infty \\ &= \frac{GM}{r} = W_{\text{kin}}(r) \\ &= \frac{1}{2}v_{\text{esc}}^2(r). \end{aligned} \tag{1}$$

Here, G is Newton's constant, M is the total mass of the spherical body, v_{esc} is the escape velocity that depends on r (radial distance from the center, measured as circumference divided by 2π), and W_{kin} is the kinetic energy per unit mass of the test body in free radial fall that started at a very distant position outside of the spherical body. It must be equal to the magnitude of the potential energy of the test body.

Rearranging Eq. (1) gives the escape velocity according to Newton's physics

$$v_{\rm esc}^2(r) = \frac{dr^2}{dt^2} = \frac{2GM}{r} = \frac{c^2 r_s}{r}. \tag{2}$$

Here, r_s ($=2GM/c^2$) is the Schwarzschild radius (where the escape velocity is c).

The *relativistic* escape velocity $v_{\rm rel}$ according to the outer Schwarzschild solution, that is, according to

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right)dt^2 - \frac{1}{c^2\left(1 - \frac{r_s}{r}\right)}dr^2 - \frac{r^2}{c^2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{3}$$

is equal to

$$v_{\rm rel}^2(r) = \frac{dr^2}{dt^2} = \frac{c^2 r_s}{r}\left(1 - \frac{r_s}{r}\right)^2. \tag{4}$$

The velocity of a freely falling/rising observer Alice—measured in coordinate time and space of a distant observer Bob who is at rest in his frame of reference—is reduced by two dimensionless factors (in comparison with Newton's physics): The first factor gives consideration to the time dilation in a gravity field, and the second factor gives consideration to the shrinking of stationary, radially oriented meter sticks in a gravity field. In a radial fall/rise, the differentials $d\varphi$ and $d\theta$ are zero.

From Eq. (4), we get

$$\Delta t = \int_{r(t)=r}^{r(t)\to\infty} dt = \int_{r=r}^{r\to\infty} \frac{1}{c\frac{\sqrt{r_s}}{\sqrt{r}}\left(1 - \frac{r_s}{r}\right)}dr = \frac{1}{c}\int_{r=r}^{\infty} \frac{\sqrt{\frac{r}{r_s}}}{1 - \frac{r_s}{r}}dr = \pm\frac{r_s}{c}\left[\frac{\left(\frac{6r_s}{r} + 2\right)\left|\left(\frac{r}{r_s}\right)^{3/2}\right|}{3} - \ln\left(\left|\sqrt{\frac{r_s}{r}} + 1\right|\right) + \ln\left(\left|\sqrt{\frac{r_s}{r}} - 1\right|\right)\right]_{r=r}^{r\to\infty}. \tag{5}$$

In order to determine the time interval Δt for the whole trip (free fall followed by free rise, and back again), the right-hand side of Eq. (5) must be multiplied by the factor 4.

III. DERIVING THE EQUATION OF FREE FALL/RISE WITH "PROPER TIME" TAU OF THE FALLING/RISING OBSERVER AS THE DEPENDENT VARIABLE

Next, we will determine the proper time τ, that is the time measured by Alice who is in free fall/rise.

From Eqs. (3) and (4), we get

$$\frac{d\tau^2}{dt^2} = \left(1 - \frac{r_s}{r}\right) - \frac{1}{c^2\left(1 - \frac{r_s}{r}\right)}\frac{dr^2}{dt^2} = \left(1 - \frac{r_s}{r}\right) - \frac{1}{c^2\left(1 - \frac{r_s}{r}\right)}\frac{c^2 r_s}{r}\left(1 - \frac{r_s}{r}\right)^2 = \left(1 - \frac{r_s}{r}\right)^2, \tag{6}$$

or

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right)^2 dt^2. \tag{7}$$

From Eqs. (7) and (4), we obtain

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right)^2 \frac{1}{\frac{c^2 r_s}{r}\left(1 - \frac{r_s}{r}\right)^2}dr^2 = \frac{r}{c^2 r_s}dr^2 \tag{8}$$

or

$$\Delta\tau = \int_{r(\tau)=r_1}^{r(\tau)\to\infty} d\tau = \int_{r_1}^{\infty}\sqrt{\frac{r}{c^2 r_s}}dr = \left[\frac{2}{3c\sqrt{r_s}}r^{3/2}\right]_{r=r_1}^{r\to\infty} = \left[\frac{2r_s}{3c}\left(\frac{r}{r_s}\right)^{3/2}\right]_{r=r_1}^{r\to\infty} \tag{9}$$

(see Ref. 1, Sec. 25.5, Eq. (25.38), 1st line, p. 667).

IV. THE RATIO OF Δt AND $\Delta\tau$

The diameter r_0 of the spherical body shall be much, much smaller than r_s and is thus approaching zero. For Alice's free rise/fall that begins/ends at $r=0$ and hence at $t=\tau=0$ [the square bracket on the right-hand side of Eq. (5) is zero for $r=0$, and so is the square bracket on the right-hand side of Eq. (9)], we then get from Eqs. (5) and (9)

$$\lim_{r\to\infty}\frac{\Delta t}{\Delta\tau} = \lim_{r\to\infty}\frac{\frac{r_s}{c}\left[\frac{\left(\frac{6r_s}{r} + 2\right)\left|\left(\frac{r}{r_s}\right)^{3/2}\right|}{3} - \ln\left(\left|\sqrt{\frac{r_s}{r}} + 1\right|\right) + \ln\left(\left|\sqrt{\frac{r_s}{r}} - 1\right|\right)\right]}{\frac{2r_s}{3c}\left|\left(\frac{r}{r_s}\right)^{3/2}\right|} = 1. \tag{10}$$

Reference 1, Sec. 25.5, Eq. (25.38), p. 667, would, for far-away starting points of a radial fall, have come to the same result as presented in Eq. (10), if it had been realized that the quotient

$$\operatorname*{limes}_{r\to\infty}\frac{\frac{t}{2M}}{\frac{\tau}{2M}}=\operatorname*{limes}_{r\to\infty}\frac{-\frac{2}{3}\left(\frac{r}{2M}\right)^{3/2}-2\left(\frac{r}{2M}\right)^{1/2}+\ln\frac{\left(\frac{r}{2M}\right)^{1/2}+1}{\left(\frac{r}{2M}\right)^{1/2}-1}}{-\frac{2}{3}\left(\frac{r}{2M}\right)^{3/2}}=1$$

of their expressions for t and τ – appearing in first and second lines of their Eq. (25.38)–approaches unity for very large r.

At $r = r_s$, the integrand in Eq. (5) flips its sign when Eq. (5) is applied to the "Black-and-White-Hole." One should note that neither the singularity at $r = r_s$ nor the singularity at $r = 0$ prevents Eq. (5) from working [and yielding Eq. (10)].

However, it should be noted that

$$\lim_{r\to\infty}(\Delta t-\Delta\tau)$$

does not exist. This can be considered as a result of the fact that the Schwarzschild metric is written in coordinate time and space (t, r) of a (stationary) distant observer supposed to be outside of the gravity field, though such a position does not exist: Gravity does not shrink to absolute zero even at large r. Hence, the Schwarzschild metric contains a small error, which surfaces when temporal intervals are very long. In other words: Bob's true time interval between the two events is a little shorter than Δt, and, given that the gravity he feels is not absolute zero, is even a little shorter than $\Delta\tau$ (measured by Alice).

V. "$(\Delta T)/(\Delta\tau) = 1$" AS A CONSEQUENCE OF THE RELATIVITY PRINCIPLE

The approximate equality of $\Delta\tau$ and Δt as expressed in (10) is not a coincidence, but is a requirement of the relativity principle

The r-geodesic, that is

$$\frac{d^2R}{d\tau^2}+\Gamma^r_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}=0 \tag{11}$$

turns into Newton's law of gravitation (valid for a local observer in the gravity field whose time is τ) in case the Schwarzschild metric is used for a determination of the metric tensor g that is hidden the Christoffel symbol. (It can be left undecided whether or not R, now defined as radial distance measured by laying radially oriented meter sticks end to end, should better be replaced by r, that is circumference divided by 2π.) That is to say: The antiradial acceleration described by Newton's law, and experienced by a local observer in the gravity field (Alice) with respect to a system of spatial coordinates, in which the central spherical body is at rest, is nothing but the result of a local curvature of spacetime. In other words: Since freely falling Alice, whose trajectory in spacetime is determined by gravitation (and nothing else), is traveling along a r-geodesic, she does not experience any true force at all.

Consequently, she is perfectly entitled to say that she is permanently at rest while other things around her are in accelerated motion. The local speed of light—measured at the position where she is permanently at rest (in her own frame of reference)—is c.

Moreover, at her stationary position in space (stationary with respect to her own frame of reference), the first derivative of the local speed of light with respect to Alice's spatial coordinate R is zero: Due to the effect of tidal forces, gravity is no longer zero for Alice at positions some radial distance away from her. That gravity may either reduce or increase the speed of light as seen from Alice's position. But since the directions of those gravitational forces, when moving in the positive and negative radial directions, are opposing each other, the situation is qualitatively symmetrical for Alice, so that the speed of light at Alice's position must be an extremum, that is, either a maximum or a minimum. But this is just another way of saying that the first derivative of the speed of light with respect to R is zero at Alice's position (in Alice's frame of reference).

In short: Freely falling reference systems (like Alice's) are inertial systems—if they are not too large.

(See Ref. 2, Chap. 8, p. 177: "As a result, he [Einstein] defined this type of reference frame which we can call a free-fall frame as the only valid inertial frame in the theory of general relativity. He considered these frames to be special because they would be the only ones in which all of the physical laws that he considered to be correct would be valid. In order to utilize this concept, he had to conclude that all such free-falling frames of reference were equivalent.")

In comparison, the situation of an observer (Alice's sister) who sits on the surface of a spherical body is different: Though the local speed of light measured by that observer is c also in this case—this is what the Schwarzschild metric tells us—the first derivative of the speed of light with respect to his/her spatial coordinates is different from zero. This can be regarded as a consequence of the fact that he/she is not traveling along a r-geodesic in spacetime, which, in turn, is the result of the fact that he/she is experiencing a repulsive, upward force exerted by the surface of the spherical body. That force is not the mere result of a local curvature of spacetime, but is electrostatic in nature.

In this context, the principle of locality postulates: Whatever physical effect happens to someone or something, is the result of *local* parameters, and not of parameters valid at positions far away. Hence, both Alice (who traverses a

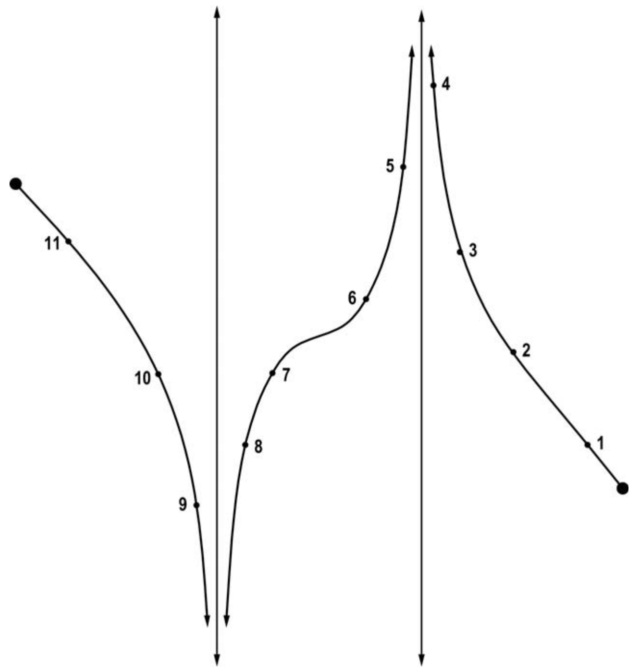

FIG. 1. Traversing a Black-and-White-Hole for which r_0, the diameter of the spherical body, is vanishingly small compared with the Schwarzschild radius r_s (represented by two vertical lines), with the world line of the traveler being determined by a single equation, namely, Eq. (5). The numbers on the graph (that represents Alice's world line) are marks of Alice's time τ. The vertical direction is Bob's time t, the horizontal direction is Bob's spatial coordinate t.

spherical body in free fall and rise) and Bob (who stays outside of the gravity field) do not only have the same right to say that they are (and have been) at rest all the time, but they can, in addition, expect that everything physical that has happened to Alice as an effect of gravity or velocity must have also happened to Bob. This is why, at the moment of reunion (at the end of the trip), the same amount of time must have elapsed for both of them.

To put it differently: Assuming that the traveling observer (Alice), if small enough, constitutes an inertial system, the relativity principle requires a reciprocity of time dilation. If the observer who had stayed outside (Bob) would find a dilation of time by a factor of 2 in the other system (Alice's) (that is, if he would find $\Delta\tau/\Delta t = 1/2$) after completion of the trip, the traveling observer Alice, whose time is τ, would, in turn, have to find that time has passed more slowly by a factor of 2 in the system of the stationary observer (Bob) than in her own (Alice's). In other words: $\Delta\tau/\Delta t$ must equal $(\Delta t'/\Delta\tau')$. The primed bracket stands for the quotient valid in Alice's frame of reference. But in our special case, this equality can only hold true if $\Delta\tau/\Delta t = \Delta\tau'/\Delta t' = 1$. In other words: If two clocks—that had been synchronized when having met each other for the first time—meet each other a second time, it is impossible (for logical reasons) that *each* of the two clocks lags behind the other.

This is quite different from the situation of an observer in the gravity field who is stationary (Alice's sister). Due to the force Alice's sister is feeling, she cannot consider herself as being in the center of an inertial system. Consequently, the time dilation with respect to that observer (Alice's sister) measured by a (stationary) distant observer (Bob) is not subject to reciprocity. This is why, from Alice's sister's perspective, Bob's time is *compressed*.

VI. THE CORRECT APPLICATION OF KRUSKAL CHARTS

Given that Eqs. (5), (9) and (10) prove that the Schwarzschild metric allows for a traverse of a "Black-and-White Hole" in free fall and rise without any difficulties (even the relativity principle is observed), why shouldn't we think that ALL Black Holes are also White Holes?

Kruskal-charts can hardly be an obstacle to saying "Yes, we can." On a Kruskal-chart, every single event—if defined by a single r-coordinate and a single t-coordinate—is represented by two points (and not just one point), with the two points sitting in two different quadrants.

Equations (5), (9), and (10) that describe Alice's free rise permit only one answer to the question as to what this doubling of event-points is all about: Quadrant II of the Kruskal-chart can only be used for ingoing traffic, but not for outgoing traffic. For outgoing traffic, a different quadrant must be used. In that different quadrant, any outgoing light pulse (moving at 45°) that originates in the interior of the Schwarzschild horizon does NOT end up at the singularity (that is, at $r = 0$), but crosses the Schwarzschild horizon into outer space. So does freely rising Alice.

Since the equality of Δt and $\Delta\tau$ (as expressed in Eq. 10) is proof of the fact that Eq. (5), despite the singularity at $r = r_s$, gives a true description of Alice's free rise across the Schwarzschild radius (and is not just an artefact), quadrant II of the Kruskal chart is barred from being used for a representation of Alice's free rise (as a dashed line in Fig. 2).

VII. THE ASYMMETRY BETWEEN BLACK HOLES AND WHITE HOLES: WHITE HOLES AS WEAKLY, BUT NOT HIGHLY IMPROBABLE OCCURRENCES

Despite of what has been said above, there seems to be an asymmetry between Black Holes and White Holes insofar as Black Holes seem to be much more abundant than White Holes. Why is that?

As is commonly accepted, strictly irreversible processes, i.e., processes that are forbidden to occur (by laws of nature) in reverse order, do not exist. Let this recognition be called the "reversibility statement." It applies to the falling of an object into a Black Hole as well. That is to say: Given an object may fall into a Black Hole, the reverse, that is the escape from a Black Hole, cannot be strictly forbidden by laws of nature.

On the microscopic level, there is not even a *preference* for a temporal direction. That is to say: If a short video of a microscopic process were shown, one could not tell whether or not the video is run in reverse. On the macroscopic level, things seem to be of a different kind. Differences in temperature within a gas enclosed in a chamber vanish, but the reverse has never been observed. That is to say: A video showing the building up of temperature differences in a gas of formerly homogenous temperature can clearly be judged as being a video that is run in reverse, and not in the time order in which the process was observed.

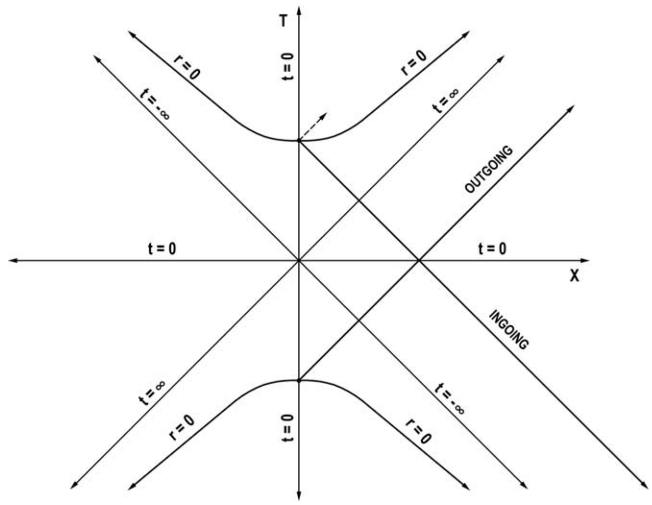

FIG. 2. Alice's world line (as depicted in Fig. 1) shown by a Kruskal-chart. According to the common misinterpretation of Kruskal-charts, the outgoing leg of the trip is represented by the short, dashed line (originating at the **upper** $t=0$, $r=0$ situated in quadrant II) that leads to nowhere. When utilizing the Kruskal-chart correctly, the outgoing leg is represented by a diagonal line that originates at the **lower** $t=0$, $r=0$ situated in quadrant IV. For reason of simplicity, the two legs are shown as straight lines (as if Alice were a light pulse), and not (as it should be) as lines that are slight curves.

As is commonly known, this temporal asymmetry on the macroscopic level is accounted for by the fact that the macroscopic state of an almost homogeneous temperature of a gas comprises many more microstates than the competing macrostate of an inhomogeneity of temperature within the gas does. Hence, the building up of a temperature difference in a gas of formerly homogeneous temperature is not considered as being physically impossible, but highly improbable. Speaking of a "highly improbable macrostate" thus means: The macrostate comprises far less (equally probable) microstates than a *competing* macrostate does.

The common (but erroneous) view on Black Holes is quite similar. If a video were shown in which an object or signal is leaving the interior of a Black Hole, common opinion would have it that the event is shown in reverse, and not in the time order in which it was observed. But how can this conviction be justified (given the fact that all microscopic processes, when shown in a video, cannot be rated in that manner, but can be presented in either temporal order)? According to the common (but erroneous) view, the case of an object that leaves the interior of a Black Hole does not resemble a microscopic process, but resembles the behavior of an ensemble of molecules of a gas whose temperature was homogeneous to start with, and in which differences in temperature are evolving over time. That is to say: Common opinion does not rule out that objects may leave the interior of a Black Hole, but consider such an event as a highly improbable macrostate.

To clarify: Whenever an object manages to cross the Schwarzschild horizon from the interior to the exterior, we speak of a "White Hole." Consequently, we speak of a "Black Hole" whenever an object crosses the Schwarzschild horizon the other way, that is, from the exterior into the interior. Hence, Black Holes and White Holes need not be different stellar objects, but can be names for different aspects of one and the same stellar object.

But is it justified to say that an object which is crossing the Schwarzschild radius from the interior to the exterior constitutes a macrostate which comprises far less microstates than the crossing of the Schwarzschild horizon from the exterior to the interior does? The answer is clearly in the negative. Both macrostates comprise one single microstate each, and not more. Given that there are only two objects in the game, which can be well distinguished from each other (the massive object and the tiny, infalling/outgoing test body), the distinction between microstates and macrostates makes no sense.

The common (but erroneous) opinion is thus facing a dilemma: It regards White Holes as highly improbable in the sense described, but cannot justify that assessment. On the contrary:

A "high improbability" for White Holes can surely be excluded, since the term does not apply to White Holes. What we are thus left with is the recognition that the "reversibility statement" *does* apply, whereas the statement of a high improbability does not. That is to say: Even without any calculations, the fallacy of the common interpretation of Kruskal-charts, according to which (practically) all outward-directed trips that originate within the Schwarzschild radius are doomed to end up at the center of the sphere, becomes obvious.

To wrap it up: White Holes are neither physically impossible, nor highly improbable. The equations displayed above do not show any preference for inward motions over outward motions of test-bodies; nor do Kruskal charts. In other words: General Relativity adheres to the reversibility statement. Moreover, the distinction between microstates and macrostates that could give rise to the rating of "highly probable" and "highly improbable" does not apply here. We can thus exclude that escaping from the interior of a Black Hole is a highly improbable event.

The only difference between Black Holes and White Holes is the following: As with ordinary stellar objects like planets (that do not have a Schwarzschild horizon), the phenomenon of *accretion* (caused by gravitational attraction) is responsible for the fact that far more small objects find their way from space into close orbits around big stellar objects (or onto their surfaces) than are ejected from close orbits around big stellar objects (or from their surfaces) into space. Even more generally speaking: Due to the attractive force of gravity, objects in space that attract each other are far more abundant than objects that repel each other. That is to say: In comparison with Black Holes, White Holes are *weakly* improbable (when it comes to observing stellar objects that have a Schwarzschild horizon).

VIII. THE RELATIVITY OF EARLIER/LATER FOR TIMELIKE EVENTS LIKE BIRTH AND DEATH OF A PERSON

Despite a corrected conception of Black Holes—according to which all Black Holes can also be White Holes—

Black-and-White Holes have not lost their role of being special. On the contrary: The relativity of an ordering of earlier-later, which, according to Special Relativity, applies to causally unrelated (spacelike) events only (occurring far away from each other), is now extended to causally related (timelike) events, for instance, the beats of Alice's heart when she is inside the Schwarzschild radius. Thereby the "relativizing" of a temporal ordering earlier/later and hence of time is made complete: The death of a person does not occur after her birth in all frames of reference; instead, there are frames in which the death is the beginning of a person's life, so that her birth is the end.

[1]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, CA, 1973).
[2]P. Graneau, and N. Graneau, *In the Grip of the Distant Universe – The Science of Inertia* (World Scientific, Hackensack, NJ, 2006).