The energy density of the gravitational field

Andreas Trupp[a)]

(Received 14 August 2019; accepted 14 November 2019; published online 12 December 2019)

Abstract: It is shown that 100-year-old, conflicting ideas on the positive or negative energy of the gravity field collide with the principle of local conservation of energy. A scrutiny of the Schwarzschild metric, carried out with a different method than that applied by E. Schrödinger but completed with a similar result, reconfirms that the gravity field holds no energy at all, with that recognition being tacitly acknowledged by Misner, Thorne, and Wheeler in 1973. Given that it does not hold any energy, it cannot, by definition, be qualified as a force-field. Given that it is not a force-field, it is capable of being completely transformed away even in the rigid reference-frame of a distant observer outside of the field. Contrary to what (early) Einstein believed, this can (and must) be achieved by the concept of "flowing spaces" that was introduced by elder Einstein himself in 1952. It is shown that this concept leads to empirical consequences. Moreover, the energy of the gravity field is necessarily replaced by an inexhaustible "dark energy," which flows into any massive object (including Newton's apple) whenever, after a free fall, it is being decelerated. Thereby Schrödinger's vision of "new foundations" of the energy conservation principle (as a consequence of his recognition that the gravity field holds no energy) is coming true. Because of the absence of any gravitational field lines that originate from that energy, the (main) seat of this dark energy cannot be in three-dimensional space, but must sit at a location separated from ordinary space by a short distance in a direction perpendicular to all three ordinary spatial directions. © *2019 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-32.4.484]

Résumé: On montre que des idées vieilles de 100 ans et contradictoires sur l'énergie positive ou négative du champ de gravité entrent en conflit avec le principe de conservation locale de l'énergie. Un examen de la métrique de Schwarzschild, effectué avec une méthode différente de celle appliquée par E. Schrödinger mais complété avec un résultat similaire, confirme que le champ de gravité ne contient aucune énergie, cette reconnaissance étant tacitement reconnue par Misner, Thorne et Wheeler en 1973. Étant donné qu'il ne contient aucune énergie, il ne peut, par définition, être qualifié de champ de force. Étant donné qu'il ne s'agit pas d'un champ de force, il peut être complètement transformé même dans le cadre rigide de référence d'un observateur éloigné en dehors du champ. Contrairement à ce qu'Einstein croyait (au début), cela peut (et doit) être réalisé par le concept d ' "espaces fluides" qui a été introduit par l'ancien Einstein lui-même en 1952. Il est démontré que ce concept entraîne des conséquences empiriques. De plus, l'énergie du champ de gravité est nécessairement remplacée par une inépuisable "énergie sombre", qui s'écoule dans n'importe quel objet massif (y compris la pomme de Newton) chaque fois que, après une chute libre, elle est décélérée. Ainsi, la vision de Schrödinger de "nouvelles bases" du principe de conservation de l'énergie (en raison de sa reconnaissance que le champ de gravité ne détient pas d'énergie) se réalise. En raison de l'absence de toute ligne de champ gravitationnel qui provient de cette énergie, le siège (principal) de cette "énergie sombre" ne peut pas être dans l'espace tridimensionnel, mais doit s'asseoir à un endroit séparé de l'espace ordinaire par une courte distance dans une direction perpendiculaire aux trois directions spatiales ordinaires.

Key words: Energy of the Gravitational Field; General Relativity; Dark Energy; Fourth Spatial Dimension; Time Reversal.

I. INTRODUCTION

Force fields are characterized by their property of being capable of transferring energy to a body they act on, at the expense of energy previously stored in that field.

The electric field serves as a perfect example. An electrically charged sphere that finds itself between the plates of a charged parallel-plate-capacitor is accelerated by the field and is thereby picking up kinetic energy. During this process, the Poynting-vector field $E \times B$ shows a flow of energy from the field that surrounds the sphere into that sphere. The sphere is a sink, and the space around the sphere is a source of that energy flow.

Moreover, since the electric field between the capacitor plates holds energy, it has mass.

Suppose we would find that, contrary to our assumption, the electric field between the two capacitor plates would have no (inert and heavy) mass. On the basis of the principle

[a)]Permanent address: Kappenberger Damm 27, 48151 Münster, Germany; andreas@andreastrupp.com

of equivalence of mass and energy, we would have to come to the conclusion that it holds no energy. Moreover, given it holds no energy, it cannot be a force field. For in order to be a force field, it would, by definition, have to be in possession of energy. That energy is transferred to the sphere, where it is converted into kinetic energy. Without any energy contents, the electric field would be the field of a pseudo force, and the kinetic energy of the sphere would have to be the result of a tapping of a hidden energy reservoir

Let us now switch from the electric to the gravitational field. Unfortunately, General Relativity does not provide a tool analogous to the Poynting vector that would make the flow of energy visible that enters Newton's apple when it is falling from the tree. Even worse, there is no consensus on how the energy density of the gravitational field can be quantified. An analogy with the electric field, according to which the energy density would be proportional to the strength of the local gravitational field (force per kg) squared, and would thus have a positive numerical value, leads to an apparent conflict with the principle of conservation of energy, and is therefore dismissed by modern textbooks. Though some authors postulate the energy density of the gravitational field as being proportional to the negative square of the strength of the gravitational field, this postulate was rejected by Einstein for good reasons.

In this article, the energy density of the gravitational field (if there is any) shall be determined once and for all.

II. METHODS

No other methods than logical and mathematical conclusions are applied. The relativity principle, Einstein's field equation, the Schwarzschild solution of Einstein's field equation, the equivalence of mass and energy (as postulated by Special Relativity), the equation of a geodesic, and the local principle of conservation of energy are used as starting-points.

III. RESULTS

A. The quest for a determination of the energy density of the gravity field

a) Einstein's field equation of General Relativity,[1,b)] that is,

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G\, T^{\mu\nu} \tag{1}$$

presupposes the vanishing of the covariant divergence of the "ordinary" energy-momentum tensor T that comprises energy and momentum of ordinary matter M (but not of matter or momentum ascribed to a gravitational field). R denotes the Ricci-Tensor, g denotes the metric tensor, and G denotes Newton's gravitational constant. The indices μ and ν run from 0 to 3, and represent the four components (one is temporal and three are spatial), that is x^0, x^1, x^2, x^3, of a four-dimensional vector (of which there are four associated with each 4×4 tensor).

Proof: The covariant divergence of the left-hand side of the equation is zero as a mathematical necessity. (A detailed proof is omitted here.) Consequently, the covariant divergence of the right-hand side of the equation, too, must vanish.We thus get

$$0 = \nabla_\mu T^{\mu\nu} = \delta_\mu T^{\mu\nu} + \Gamma^{\mu}_{\alpha\mu} T^{\alpha\nu} + \Gamma^{\nu}_{\alpha\mu} T^{\mu\alpha}. \tag{1a}$$

The vanishing of the ordinary divergence of the energy-momentum-tensor T [that is, the vanishing of the first summand on the right-hand side of Eq. 1(a)] seems to be an expression of the principle of conservation of energy. In order to visualize this, we choose a reference frame in which no mass is changing its spatial position over time. Consequently, the energy-momentum-tensor T has one single non-zero component only (T^{00}), that is, a component in the x^0 (temporal)-direction. Given that the mass is spread out in space, the energy-momentum tensor field (in that reference frame) is a vector field whose direction is parallel to the x^0-axis, and whose magnitude is that of the local mass density. If that vector-field had an ordinary divergence different from zero, the principle of conservation of energy (=mass) would seemingly be violated, as mass (=energy) would appear or disappear into nothingness. But what about the energy and mass ascribed to the gravitational field? As A. Einstein put it:[c)]

> "It must be remembered that besides the energy density of the matter there must also be given an energy density of the gravitational field, so that there can be no talk of principles of conservation of energy and momentum alone."

As a consequence, Einstein postulated the following equation as an expression of the principle of conservation of energy:[2,d)]

$$0 = \frac{\delta T^{\alpha}_{\sigma}}{\delta x_\alpha} + \Gamma_{\sigma\beta} T^{\alpha}_{\beta} = \frac{\delta T^{\alpha}_{\sigma}}{\delta x_\alpha} + \frac{1}{2} \frac{\delta g^{\mu\nu}}{\delta x_\sigma} T_{\mu\nu}. \tag{2}$$

Einstein's Equation (2) is equivalent to the more familiar Eq. (1a): Multiplication of both sides of Eq. (1) by the metric tensor $g_{\mu k}$ changes the components of the tensors from contravariant components to mixed components that can be given different letters thereafter. Formulating the covariant divergence of the new T—which must still vanish—leads to Eq. (2). To recapitulate: Equation (2) is in accordance with Eq. (1), as it states that the covariant divergence of T (not the ordinary one) is zero, which is mathematically presupposed by Eq. (1). The covariant divergence is distinguished from the ordinary divergence by a summand that contains the Christoffel symbol.

b)See its formulation by A. Einstein, *The Meaning of Relativity*, 5th ed. (Princeton U.P., 1956), therein: "The general theory," p. 84, Eq. (96).

c)*The Meaning of Relativity*, 5th ed. (Princeton U.P., 1956), therein: "The general theory," p. 83.

d)"The foundation of the general theory of relativity," translated from "Die Grundlage der allgemeinen Relativitätstheorie," Ann. Phys. **354**, 769 (1916); in: A. Einstein, H. A. Lorentz, and H. Minkowski, *The Principle of Relativity* (Dover Publ., 1952), § 18, Eqs. (57) and (57a), p. 151.

One is getting a deeper understanding of Eq. (2) by imagining a test body that, from the perspective of a faraway observer, is "gathering speed" in a gravity field. In such a case, T is of a kind that makes Eq. (2) an expression of a geodesic along which the test body is coasting when left to itself.[3,e)] If the gradient of the components of the metric tensor g in Eq. (2) is not vanishing, the ordinary divergence of T is different from zero. The nonvanishing, ordinary divergence of T is compensated by the last summand on the very right hand side of Eq. (2). In Einstein's view, kinetic energy of the falling body is thus converted into energy of the gravitational field, or vice versa:[f)]

> "Physically, the occurrence of the second term on the left-hand side [this term of Einstein's equals the last term on the very right-hand side of our Equation 2] shows that laws of conservation of momentum and energy do not apply in the strict sense for matter alone, or else that they apply only when the g are constant, i.e. when the field intensities of gravitation vanish. This second term is an expression for momentum, and for energy, as transferred per unit of volume and time from the gravitational field to matter."

Since the second summand on the very right side of Eq. (2) can be different from zero, it follows from Eq. (2) that the ordinary divergence of T (first summand) may be different from zero. This is why the tensor T cannot represent the total energy-momentum density of the system, but only a part of it. Otherwise the principle of conservation of energy and momentum would be violated.

From Eq. (2), one can derive[4,g)]

$$0 = \frac{\delta\left(T^{\sigma}_{\mu} + t^{\sigma}_{\mu}\right)}{\delta x_{\sigma}}. \tag{3}$$

According to Einstein, t is an expression of the density of "momenergy" ascribed to the gravitational field. Einstein thus asserts that the ordinary divergence of T plus the ordinary divergence of t amount to zero.

If a reference frame is chosen in which all components of the metric tensor are constants over a small region of space, and in which (by arrangement) the only nonvanishing component of T inside a material body is $T^0{}_0$ (mass at permanent rest), the ordinary divergence of T is zero inside and outside of the material body. (One has to keep in mind that a tensor with mixed indices has absorbed the metric tensor, hence the vanishing of the ordinary divergence of T is guaranteed only in case all components of the metric tensor are constants.) Then, however, the ordinary divergence of t, too, must be zero according to Eq. (3).

But does this allow the determination of the components of t, especially of the magnitude of $t^0{}_0$? The answer is in the negative.

(b) After all, an integral which is founded on Eq. (3), that is (here the fourth component is the timelike component)

$$\int_V \left(T^4_i + t^4_i\right) dx^1 dx^2 dx^3 = const \tag{4}$$

has (under certain restrictions) the same value in any reference frame.[h)] That is to say: The sum of all momenergies of bodies and gravitational fields is not only constant over time, but is also the same in any frame of reference. For comparison, in Special Relativity, the momenergy of a body has the same magnitude in any frame of reference, and is equal to the mass of the body in its own rest frame. In General Relativity, it is the sum of the momenergies of bodies and gravitational fields that apparently replaces the momenergies of bodies alone.

Given the indeterminateness of the magnitude of the components of t, it is no longer a surprise to find that the postulated energy-momentum density t of the gravitational field, and also one of its components, that is the energy density of the gravitational field $t^0{}_0$ of the gravitational field, can be made to vanish by means of an appropriate choice of "flat" coordinates on the basis of Eq. (3).[i)]

Even more: One- and the same observer arrive at completely different results for t and hence for the energy density of the gravitational field, depending on which coordinate system he or she is using.[5,j)]

In the face of this recognition, W. Pauli drew the following conclusion:[k)]

> "According to this, one cannot assign any physical meaning to the values of the t themselves, i.e. it is impossible to carry out a localization of energy and momentum in a gravitational field in a generally covariant and physically satisfactory way."

Hence, no definite, time- and space-dependent value can be attributed to the local energy density of the gravitational field [at least not on the basis of Eq. (3)].

c) This stance of Einstein's and Pauli's presents a sharp contrast to the long-standing and still popular assertion according to which the energy density of the gravitational field is proportional to the negative square of the local intensity of the gravitational field. That assertion has been backed

e)W. Pauli, *Theory of Relativity* (Dover Publ., 1981), Sec. 54, p. 158.

f)See A. Einstein, op. cit, after Eq. (57).

g)See A. Einstein, "The foundation of the general theory of relativity," § 17, Eq. (56), p. 150; A. Einstein, "*Der Energiesatz in der allgemeinen Relativitätstheorie*," Sitzungsberichte der Preußischen Akademie der Wissenschaften **1**, 448–459 (1918).

h)See W. Pauli, *Theory of Relativity* (Dover Publ., 1981), Sec. 61, Eq. (447), p. 176; A. Einstein, "*Der Energiesatz in der allgemeinen Relativitätstheorie*," Sitzungsberichte der Preußischen Akademie der Wissenschaften **1**, 448–459 (1918), Eq. (25).

i)See W. Pauli, *Theory of Relativity* (DoverPubl., 1981), Sec. 61, p.176: "Since these quantities do not depend on the derivatives of the g higher than the first, we can conclude immediately that they can be made to vanish at an arbitrarily prescribed world point for a suitable choice of the coordinate system (geodesic reference system)."

j)See H. Bauer, "Über die Energiekomponenten des Gravitationsfeldes," Physikalische Zeitschrift **19**, 163–165 (1918): "As a concluding remark, we may state that the 'energy components' t have nothing to do with the existence of a gravitational field, but depend on the choice of coordinates only…."

k)W. Pauli, *Theory of Relativity* (Dover Publ., 1981), Sec. 61, p. 177.

by the following argument: Since General Relativity can be expected to be almost indistinguishable from Newton's physics when it comes to objects like the solar system, and since Newtonian physics has been understood as equaling the negative potential energy and the energy of the gravitational field, one might be inclined to assume that the energy density near gravitating masses has to be negative also in General Relativity. See W. Pauli:[l)]

> "In the earlier field theories of gravitation already, it was the sign of the energy density of the gravitational field which had led to difficulties. In spite of these difficulties it would, on physical grounds, be hard to abandon the requirement that an analogue to the energy- and momentum-integrals of Newtonian theory should exist."

This led *T. Levi-Civita* and *H. A. Lorentz* to the belief that the sum of positive energy of matter and (postulated) negative energy of the gravitational field is always zero in a closed system. Even today, this belief is very popular among physicists.[6,7,m)]

Einstein rejected this assertion categorically,[8,n)] because it would then be possible "for a material system to vanish into nothing without leaving a trace."

Moreover, those authors who postulate that the energy density of the gravitational field is proportional to the negative square of its magnitude do not give any clue as to how their postulate can be subject to empirical testing. Does a negative sign of the gravitational energy density imply that the mass that corresponds to this energy is negative, too? If so, would we have to expect that the gravitational force exerted by that mass on ordinary mass is repulsive?

d) Despite the impossibility of giving the energy density and the momentum density of the gravity field a definite value at a certain place and time, Eq. (4) is meaningful to Pauli and Einstein, since that equation appears to make it possible to[o)] "calculate the change in the material energy of a closed system in a simple fashion." For Pauli and Einstein, the principle of conservation of energy is observed in General Relativity by means of Eq. (4). Einstein put it the following way:[p)]

> "Contrary to our present thinking habits, we thus arrive at attributing more reality value to an integral than to its differentials."

But such a reasoning can be convincing only in case the principle of conservation of energy (and also the principle of conservation of momentum) is not understood as a *local* principle. Otherwise, that is, if one does understand the energy principle as a local one (saying that energy can never simply appear or disappear at some location, but can only flow into that location or out of it), then the two equations (3) and (4)—given that the local energy density is indeterminate in any frame of reference—are not sufficient to guarantee the principle of energy conservation in General Relativity.

This insufficiency surfaces in an obvious manner in Einstein's short paper "Notiz zu E. Schrödingers Arbeit 'Die Energiekomponenten des Gravitationsfeldes'":[9,q)]

> "There can well be gravitational fields without tensions and without energy density."

But when a test body is "gathering speed" in a gravitational field that is supposed to be a force field, energy has to flow into the test body from its immediate surroundings. This is similar to the case of an electric charge set into accelerated motion by an electric field, where the energy-flow (from the adjacent electric field into the accelerating charge) is made "visible" by the Poynting vector. Presuming (for a short while at least) that no energy reservoir other than the gravitational field is available, energy has to flow from the gravitational field into the test body. Then, however, the gravitational field (assumed to be a force field) cannot have zero energy density at this location (contrary to Einstein's view).

This recognition is commonly accepted.[r)]

To put it the other way around: If the gravity field is void of energy, it cannot be a force field. Otherwise the principle of local conservation of energy would be violated. It even makes perfect sense to define a force field as a field whose energy is transferred to an accelerated body. Hence, if a field has no energy, it cannot be a force field by definition.

B. The absence of any heavy mass (and hence of any energy density) of a gravitational field

a) If a gravity field were in possession of energy, it would have to have heavy mass. As A. Einstein put it:[s)]

[l)] *Theory of Relativity* (Dover Publ., 1981), Sec. 61, p. 176.

[m)] See only Brian Greene, "The Hidden Reality, Parallel Universes and the Deep Laws of the Cosmos" (2011), Note 9 to pages 65–70, Chap. 3 , p. 381: "The gravitational field can supply the particles with such positive energy because gravity can draw down its own energy reserve, which becomes arbitrarily negative in the process: the closer the particles approach each other, the more negative the gravitational energy becomes (equivalently, the more positive the energy you'd need to inject to overcome the force of gravity and separate the particles once again). Gravity is thus like a bank that has a bottomless credit line and so can lend endless amounts of money; the gravitational field can supply endless amounts of energy because it sown energy can become ever more negative. And that's the energy source that inflationary expansion taps."; see also Alex Vilenkin, "Many Worlds in One—The Search for Other Universes," (2006), Part I 1, pp. 11/12: "So the energy of the inflating chunk must also have grown by a collossalfactor, while energy conservation requires that it should remain constant. The paradox disappears if one remembers to include the contribution to the energy due to gravity. It has long been known that gravitational energy is always negative. This fact did not appear very important, but now it suddenly acquired a cosmic significance. As the positive energy of matter grows, it is balanced by the growing negative gravitational energy. The total energy remains constant, as demanded by the conservation law."

[n)] See his paper: "Über Gravitationswellen," Sitzungsberichte der königlich preußischen Akademie der Wissenschaften (1918), Semi-Volume 1, pp. 154–167 [167].

[o)] See Pauli, *Theory of Relativity* (Dover Publ., 1981), Section 61, p. 177.

[p)] A. Einstein, "Der Energiesatz in der allgemeinen Relativitätstheorie", op. cit., p. 452.

[q)] Physikalische Zeitschrift **19**, 115/116 (1918).

[r)] As an example, see the entry "vacuum solutions" in the English Wikipedia: "Since $T^{ab} = 0$ in a vacuum region, it might seem that according to general relativity, vacuum regions must contain no energy. But the gravitational field can do work, so we must expect the gravitational field itself to possess energy,…"

[s)] "The foundation of the general theory of relativity," translated from "Die Grundlage der allgemeinen Relativitätstheorie," Ann. Phys. **354**, 769 (1916), in: A. Einstein, H. A. Lorentz, and H. Minkowski, *The Principle of Relativity* (Dover Publ., 1952), § 16, p. 148.

"For if we consider a complete system (e.g. the solar system), the total mass of the system, and therefore its total gravitating action as well, will depend on the total energy of the system, and therefore on the ponderable energy together with the gravitational energy."

One can, however, show that the gravity field does not have any heavy mass (or any energy). The absence of any energy of the gravity field follows from the Schwarzschild-equation (as a solution of Einstein's field equation for a non-rotating, spherical mass and its surroundings). According to the Schwarzschild-equation, the intensity of the gravitational "force" is inversely proportional to the square of the distance r (=circumference of a circle, divided by 2π) from the central spherical body, and directly proportional to its ordinary mass M, exactly the same as in Newton's law of gravitation.

Proof: The formula for a r-geodesic reads

$$\frac{d^2R}{d\tau^2} + \Gamma^1_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0. \tag{5}$$

Equation (5) simply says: The acceleration a body experiencing is equal to the magnitude of the local curvature of spacetime. While r denotes circumference of a circle around the center of the spherical mass divided by 2π, the variable R denotes a radial distance measured by laying meter sticks end-to-end. Hence, dR denotes a short difference in radial distance measured by a local observer who is at rest in the gravity field. It is the second derivative of that R (and not of r) with respect to the proper time τ of a local, stationary observer which describes the acceleration felt by this observer.

The Christoffel symbol, that is,

$$\Gamma^p_{\mu\nu} = \frac{g^{pn}}{2}\left(\frac{\delta g_{n\mu}}{\delta x^\nu} + \frac{\delta g_{n\nu}}{\delta x^\mu} - \frac{\delta g_{\mu\nu}}{\delta x^n}\right) \tag{6}$$

shall be written in full detail on the basis of the Schwarzschild metric, that is, on the basis of the metric tensor (G is Newton's constant, c is the speed of light, and M is the central mass)

$$g_{\mu\nu} = \begin{matrix} 1-\dfrac{2GM}{c^2r} & 0 & 0 & 0 \\ 0 & -\left[c^2\left(1-\dfrac{2GM}{c^2r}\right)\right]^{-1} & 0 & 0 \\ 0 & 0 & -\dfrac{r^2}{c^2} & 0 \\ 0 & 0 & 0 & -\sin^2\theta \end{matrix}\,. \tag{7}$$

GM can be replaced by $r_sc^2/2$, with r_s denoting the Schwarzschild radius, that is the special distance from the center of the spherical mass at which the escape velocity is c (speed of light) in Newtonian physics. We then get (the index 0 stands for the time t of a distant observer, the index 1 stands for r, the index 2 stands for the azimuthal angle θ, and the index 3 stands for the polar angle ϕ)

$$\begin{aligned}\Gamma^1_{00}\frac{dx^0}{d\tau}\frac{dx^0}{d\tau} &= \left[\frac{g^{10}}{2}(\dots) + \frac{g^{11}}{2}\left(\frac{\delta g_{10}}{\dots} + \frac{\delta g_{10}}{\dots} - \frac{\delta g_{00}}{\delta x_1}\right)\right.\\ &\quad \left. + \frac{g^{12}}{2}(\dots) + \frac{g^{13}}{2}(\dots)\right]\frac{dx^0}{d\tau}\frac{dx^0}{d\tau}\\ &= \frac{c^2}{2}\left(1-\frac{r_s}{r}\right)\frac{d(1-r_s/r)}{dr}\frac{dt^2}{d\tau^2}\\ &= \left(1-\frac{r_s}{r}\right)\frac{c^2r_s}{2r^2}\frac{dt^2}{d\tau^2}\end{aligned} \tag{7a}$$

and

$$\begin{aligned}\Gamma^1_{11}\frac{dx^1}{d\tau}\frac{dx^1}{d\tau} &= \left[\frac{g^{10}}{2}(\dots) + \frac{g^{11}}{2}\left(\frac{dg_{11}}{dx^1} + \frac{dg_{11}}{dx^1} - \frac{dg_{11}}{dx^1}\right)\right.\\ &\quad \left. + \frac{g^{12}}{2}(\dots) + \frac{g^{13}}{2}(\dots)\right]\frac{dx^1}{d\tau}\frac{dx^1}{d\tau}\\ &= \left[-\frac{c^2}{2}\left(1-\frac{r_s}{r}\right)\frac{d\left(-\dfrac{1}{1-r_s/r}\right)}{c^2dr}\right]\frac{dr^2}{d\tau^2}\\ &= \left[-\frac{c^2}{2}\left(1-\frac{r_s}{r}\right)\frac{1}{c^2(1-r_s/r)^2}\frac{r_s}{r^2}\right]\frac{dr^2}{d\tau^2}\\ &= -\left[\frac{r_s}{2(1-r_s/r)r^2}\right]\frac{dr^2}{d\tau^2}.\end{aligned} \tag{7b}$$

All other products of Christoffel-symbols (denoted by Γ) and their appropriate $dx/d\tau$ times $dx/d\tau$ vanish (given $d\phi/d\tau$ and $d\theta/d\tau$ are both zero).

We then get from Eqs. (5)–(7a) and (7b)

$$\begin{aligned}&\frac{d^2R}{d\tau^2} + (1-r_s/r)\frac{c^2r_s}{2r^2}\frac{dt^2}{d\tau^2}\\ &\quad + \frac{r_s}{2(1-r_s/r)^2r^3}\frac{dr^2}{d\tau^2} - \frac{r_s}{2(1-r_s/r)r^2}\frac{dr^2}{d\tau^2} = 0.\end{aligned} \tag{7c}$$

After multiplication by $d\tau^2/dt^2$, $1/r$, and $1/(1-r_s/r)$, Eq. (7c) turns into

$$\frac{d^2R}{d\tau^2}\frac{d\tau^2}{dt^2}\frac{1}{r(1-r_s/r)} = -\frac{c^2r_s}{2r^3} + \frac{r_s}{2(1-r_s/r)^2r^3}\frac{dr^2}{dt^2}. \tag{8}$$

Since, according to the Schwarzschild metric, $d\tau^2/dt^2$ times $(1-r_s/r)^{-1}$ equals unity if τ is the time of a (at least momentarily) stationary observer in the gravity field, and since dr/dt is zero with respect to a (at least momentarily) stationary object, we get for this situation from Eq. (8):

$$\frac{d^2R}{d\tau^2} = -\frac{c^2r_s}{2r^2} = -\frac{MG}{r^2} \tag{9}$$

or

$$\frac{d^2R}{dt^2} = -\frac{c^2r_s}{2r^2}\left(1-\frac{r_s}{r}\right) = -\frac{MG}{r^2}\left(1-\frac{r_s}{r}\right). \tag{10}$$

Equation (10) is identical with the result obtained by Droste in 1917.[10,t)] M is the ordinary mass of the spherical body and G is Newton's gravitational constant.

In order to make sure that the choice of d^2R (rather than d^2r) in Eq. (5) and hence in Eqs. (9) and (10) is justified, let us reconsider Eq. (8):

$$\frac{d^2R}{d\tau^2}\frac{d\tau^2}{dt^2}\frac{1}{r(1-r_s/r)}=-\frac{c^2r_s}{2r^3}+\frac{r_s}{2(1-r_s/r)^2r^3}\frac{dr^2}{dt^2}. \tag{10a}$$

This equation can be rearranged in order to present itself as follows, given that τ is the time of an observer at rest in the gravity field, dr/dt is the velocity of an object in free fall in the frame of an observer outside the gravity field, v' is the velocity of that object in the frame of the second observer who is at rest in the gravity field, and who is watching the object passing by:

$$\begin{aligned}\frac{d^2R}{d\tau^2}&=\left[-\frac{c^2r_s}{2r^2}+\frac{r_s}{2(1-r_s/r)^2r^2}\frac{dr^2}{dt^2}\right]\left(1-\frac{r_s}{r}\right)\frac{dt^2}{d\tau^2}\\&=\left[-\frac{c^2r_s}{2r^2}+\frac{(v')^2r_s}{2r^2}\right].\end{aligned} \tag{10b}$$

$(1-r_s/r)^{-2}\ dr^2/dt^2$ is equal to $(v')^2$ (radial velocities are affected both by the contraction of meter sticks and by the dilation of time), and $(1-r_s/r)\ dt^2/\mathrm{d}\tau^2$ is equal to unity.

According to Eq. (10b), an object whose radial velocity is c does not experience any more radial acceleration. Consequently, radial velocities that are below c in the beginning cannot exceed the local speed of light outside of the Schwarzschild radius. Moreover, Eq. (10b) shows that even far away from the gravitating mass, that is at a large—though not infinite—distance r from the mass, an object cannot be in possession of a velocity greater than c to start with: If the velocity v' of an object at a large distance r were greater than c, the acceleration $d^2R/d\tau^2$ would be positive, meaning that the object would be decelerating (though only slightly if v' is not much greater than c) because of a reversal of the gravitational force. But this could not be a physically valid statement. Instead, one has to conclude that velocities greater than c are physically impossible even in almost flat spacetime, that is, in Special Relativity.

The fact that, according to Eq. (10b), $d^2R/d\tau^2$ is always negative or zero has a further consequence: Imagine we would have written $d^2r/d\tau^2$ instead of $d^2R/d\tau^2$ in Eq. (5) and hence in Eq. (10b). Then Eq. (10b) would require that $d^2r/d\tau^2=(1-r_s/r)^{-1}d^2r/dt^2$ has to be always negative (given $v'^2<c^2$). But since a freely falling object, when watched from outside of the gravity field, is gathering speed only to start with, and is slowing down near the Schwarzschild radius, d^2r/dt^2 cannot always be negative [whereas $(1-r_s/r)^{-1}$ is always positive]. Instead, d^2r/dt^2 has to change sign from negative to positive somewhere between the starting point of the free fall and the Schwarzschild radius. In order to avoid this inconsistency, d^2R, and not d^2r, had to be used in Eq. (5) and in Eq. (10c).

Back to the correct equation (9). For a momentarily stationary observer in the gravity field, the gravitational acceleration $d^2R/d\tau^2$, that is, the gravitational force per unit mass of a test body, is, according to Eq. (9), directly proportional to the mass M of the gravitating body (and does not differ from the acceleration yielded by Newton's law of gravitation). For if one doubles the ordinary density and hence the ordinary mass of the gravitating body, one doubles the gravitational force.

If a heavy mass of the gravitational field appeared in the Schwarzschild-equation in a hidden manner and thus co-determined the intensity of the gravitational force, and if one regarded the energy of the gravity field as being proportional to the positive or negative square of the intensity of the gravity field, a direct proportionality between the ordinary mass M and the gravitational force [as given in Eq. (9)] would be unexplainable because of that quadratic relationship.

On top of this, Eq. (9) proves that the local gravitational force, that is the force felt by a stationary observer in the field, is the same function of r as it is according to Newton's law of gravitation. Then the density of gravitational field lines, too, is the same expression of the local gravitational force as it is according to Newton's law of gravitation. Consequently, the gravitational field lines are divergenceless (outside of the gravitating, spherical mass) according to Eq. (9). This, too, shows: The gravity field that exists in empty space does not exhibit a heavy mass. Even more: There is no additional mass (=energy) at all—that would sit inside or outside of the central body—besides the ordinary gravitating mass.

Note that the absence of any mass or energy of the gravitational field occurs despite the fact that the components of the metric tensor g shown in Eq. (7) are not all zero or unity.

From this follows (recapitulated): According to the Schwarzschild-equation and hence according to General Relativity, heavy mass only exists in the form of ordinary mass M, not in the form of mass of the gravitational field. Hence, according to Special Relativity and its principle of equivalence of mass and energy, the gravitational field cannot have any energy.

Such a result is tacitly—though not explicitly—acknowledged by C. W. Misner, K. S. Thorne, and J. A. Wheeler in their famous standard textbook on gravitation:[11,u)]

> "Not one of these properties does 'local gravitational energy-momentum' possess. There is no unique formula for it, but a multitude of quite distinct formulas. The two cited are only two among an infinity. Moreover, 'local gravitational energy-momentum' has no weight. It does not curve space. It does not serve as a source term on the right hand side of Einstein's field equations."

---

t)J. Droste, "The field of a single center in Einstein's theory of gravitation and the motion of a particle in that field," Proceedings of the Royal Netherlands Academy of Science **19 I**, 197–215 (1917), especially page 203.

u)Gravitation, 1973, Chapter 20.4: Why the energy of the gravitational field cannot be localized, p. 467.

In order to realize why the gravitational field does not curve space, consider a location outside an ordinary mass (that is, in vacuum), so that T is zero. If Eq. (1) is multiplied by the metric tensor, we get

$$0 = g_{\mu\nu}\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right) = R - 2R = -R. \tag{10c}$$

With the Ricci-scalar R and the energy-momentum tensor T being zero, the Ricci-tensor, too, has to be zero according to Eq. (1). This is true for any metric, and even applies to places in a very strong gravitational field or in a gravitational wave.

With the local gravitational energy-momentum having no weight, and given the equivalence of heavy mass and energy, the gravitational field has no energy.

b) This recognition was anticipated by E. Schrödinger.[12,v] Schrödinger combined Eq. (3) and the Schwarzschild-solution. He thereby achieved the following result:[w]

> "As mentioned above in an anticipating manner, it follows … that t vanishes everywhere (outside of the gravitating sphere) identically in all four coordinates within the chosen frame of reference. 3. This result appears to me under all circumstances of massive importance for our idea of the physical nature of the gravitational field. For we either have to dispense with qualifying t – defined by (2) – as energy components of the gravitational field; thereby, however, the importance of the 'conservation principles' (see A. Einstein I.c) would collapse, and the task would arise to secure this integral part of the foundations in a new way. – If, instead, we hold on to the terms (2), then our calculation teaches us that real gravity fields do exist (i.e., fields that cannot be 'transformed away'), with vanishing energy components, or, more precisely, energy components that can be 'transformed away'; …"

The latter of the two alternatives has to be dismissed: If there were true gravity fields, that is, gravity fields that cannot be "transformed away," these fields would, as true force-fields, have to have energy, that is, energy with components that cannot be transformed away, and would, because of the equivalence of mass and energy, have to act as a source of gravitational field lines. But the Schwarzschild solution tells us that it is only the ordinary mass and energy M that acts as a source of gravitational field lines.

C. The total absence of a true gravitational force and the indispensability of the idea of flowing spaces in General Relativity

a) Hence, the term t in Eq. (3) does *not* stand for the energy components of the gravitational field. Given that the gravitational field has no energy, there is only one explanation for the fact that an object in free fall is nevertheless gathering speed: All gravitational fields can, without exception, be transformed away by the recognition that there are space volume elements in the vicinity of a gravitating mass that are permanently emerging and flowing toward the center of the gravitating mass. A second possibility, namely, the existence of some unknown dark force that is acting on the falling object, can be dismissed for the following reason: If such a force existed, a freely falling electric point charge would, in the rest frame of that charge, have an electric field whose shape is not spherically symmetrical. But this would, as will be shown below, contradict the relativity principle.

More precisely: According to the correct interpretation of the solution of Einstein's field equation for spherical masses found by Schwarzschild (and by Droste only a short time thereafter), namely, (τ stands for the proper time of an observer who is sitting in the gravity field, t denotes the time of an observer who is at rest far away from the gravitating mass, G is Newton's gravitational constant, c is the speed of light, r denotes the "distance" between an observer and the center of the spherical, gravitating mass, with this distance r being circumference of a circle, divided by 2π; θ and ϕ are angles in the system of polar coordinates that are used)

$$\begin{aligned} d\tau^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= \left(1 - \frac{2GM}{c^2 r}\right)dt^2 - \frac{1}{c^2\left(1 - \frac{2GM}{c^2 r}\right)}dr^2 \\ &\quad - \frac{r^2}{c^2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \end{aligned} \tag{11}$$

there is no gravitational force. Instead, new space volume elements are steadily popping up in space out of nothing, so that space is permanently flowing toward the surface of the spherical mass, in order to disappear somewhere in its interior. The velocity of a volume element of space is increasing while the volume element is approaching the surface of the spherical mass. Objects floating in space take part in that accelerated flow.

Thereby all gravitational force fields are capable of being transformed away (without the introduction of flowing or streaming space, spherically symmetrical gravitational forces can undoubtedly be transformed away only locally, but not ubiquitously).

b) An illustration of the flow of space volume is provided by a modification of the Schwarzschild-metric, namely, by replacing $2MG/c^2r$ with H^2R^2/c^2, so that the metric can be applied to an expanding de-Sitter-universe characterized by a Hubble-constant H (=escape velocity, divided by distance) which is constant both in space and in time. The replacement is achieved by setting all components of the tensor T appearing in Einstein's field equation equal to zero, and by setting Einstein's additional term that appears on its left side, that is the summand that contains the cosmological constant λ as a coefficient, no longer equal to zero, but by giving λ a positive numerical value (which is proportional to H^2).

---

[v] "Die Energiekomponenten des Gravitationsfeldes," Physikalische Zeitschrift **19**, 4–7 (1918).
[w] Op. cit., pp. 6/7.

Then we get (τ is the proper time of a second observer far away from Earth and the Milky Way, t is the time of a first observer who sits on Earth, R is the distance between the two observers, measured as circumference of a circle around Earth, divided by 2π)

$$d\tau^2 = \left(1 - \frac{H^2R^2}{c^2}\right)dt^2 - \frac{1}{c^2\left(1 - \frac{H^2R^2}{c^2}\right)}dR^2 - \frac{R^2}{c^2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{12}$$

All points in space whose distance from the first observer is not too close are escaping toward the cosmic event horizon on straight lines. No force is exerted on objects, for instance, on galaxies, that sit at those points in space. Instead, the accelerated motion is brought about by a steady expansion of space, that is, by a permanent popping up of space volume elements between the galaxies.[13,x)]

With regard to the gravitational field of a spherical mass, the situation is just alike. Different from the expanding universe, though, the emerging space volume elements do not drift toward the cosmic event horizon, but to the surface of the spherical mass (or, if the spherical mass constitutes a Black Hole, to the Schwarzschild-horizon),[14,y)] in order to disappear into nothingness in the interior of the spherical mass.

Note that according to the Schwarzschild-equation, stationary, radially oriented meter sticks and stationary clocks in gravity fields behave exactly like meter sticks and clocks in Special Relativity do in case they are moving (along straight lines) at a velocity that is equal in amount to the escape velocity at the considered location in the gravity field. The straight and unaccelerated motion of Special Relativity is, in General Relativity, substituted by the permanent motion of objects—that sit at constant distance r from the center of mass—relative to the flow of space which is passing by (at a velocity that depends on radial distance r).

c) According to the famous interpretation of Einstein's field equation by J. A. Wheeler, masses in space tell space how to curve, and the curvature of space tells masses how to move. In order to realize this immediately, one should return to Eq. (2). This equation describes a geodesic, that is, the path taken by a body in space that is left to itself. In case space is curved, the partial derivatives of the components of the metric tensor g are not all zero. As a consequence, the first summand in Eq. (2), namely, the ordinary divergence ofthe energy-momentum tensor T, is different from zero. This means that, according to Eq. (2), the coasting body absorbs or gives off kinetic energy and momentum. As has just been stated, this is solely due to the fact that the derivations of the components of g in Eq. (2) are not all zero. It is for this reason that gravity is said to be nothing but curvature of space. No curvature of space (that is, all spatial derivatives of the components of g being zero), no acceleration. But in this mathematical view of gravity, not a single word is being said as to whether space inthe gravitational field is moving or is at rest, instead. However, the arguments displayed above compel us to accept that the curvature of space can result in accelerated motions of masses only if volumes of space *move* in the reference frame of a distant observer.

That is to say: Not only in Special Relativity, but also in General Relativity the contraction of meter sticks and the slowing down of clocks is nothing but the result of their motions in space. As regards General Relativity, Einstein elaborated this recognition with respect to a rotating disc. The Schwarzschild metric, according to which stationary, radially oriented meter sticks and stationary clocks in gravity fields behave exactly like meter sticks and clocks in Special Relativity do in case they are moving (along straight lines) at a velocity that is equal in amount to the escape velocity at the considered location in the gravity field, proves that this is also true for the vicinity of a spherical, gravitating mass. Here, the clocks and meter sticks do not move relative to a distant observer who sits outside of the gravity field, but relative to adjacent space that is passing by the clocks and meter sticks.

D. Empirical consequences of space that flows or streams

a) By the flow of space volumes, the gravitational force is completely and ubiquitously transformed away in the rigid reference frame of a far-away observer. One should stress the fact that it is not the curvature of space, that is, the nonzeroness of the second derivative of the metric tensor with respect to coordinates, which is ubiquitously transformed away in the reference frame of a distant observer (recall that the curvature as expressed by the Ricci-tensor is zero in vacuum in any frame of reference, and thus cannot be transformed away for logical reasons), but the gravitational force: What had been regarded as the result of an accelerating force on a massive object in Newtonian physics is now seen as the effect of an accelerated motion of the space volume that surrounds the object. Relative to that space volume, the object is not accelerated, and is hence not subject to a force. Eventidal forces felt by a radially falling body, which turn up if the size of the falling body is not small enough, and which have always been considered as being an obstruction to the total replacement of gravitational forces by an acceleration of the body's rest frame, are transformed away: These forces must now be seen as the result of the emergence of additional space volumes in the interior of the falling body.

The complete absence of any gravitational force is made evident especially when looking at electric charge that is falling in a gravity field. No electromagnetic radiation is generated. For a co-falling observer, the electrostatic field of a point charge stays spherically symmetrical.

Otherwise the famous observer in a freely falling elevator cabin could, contrary to the relativity principle according to which the co-falling observer in the elevator cabin may consider himself as being in the center of a local inertial

[x)] See A. Einstein, *Relativity – The Special and the General Theory* (Bonanza Books, 1961), Appendix IV, p. 134: "Namely, the original field equations admit a solution in which the 'world radius' depends on time (expanding space)."

[y)] See H. Reichenbach, *The Philosophy of Space and Time* (Dover Publ., 1958), § 36, Fig. 41, p. 226.

system and hence at rest, tell by means of a simple observation, namely by determining the shape of the electrostatic field of a charge at rest, whether he is falling in a gravity field, or whether he is far away from heavy masses, instead.[15,z)] If, as is required by the relativity principle, the electrostatic field stays spherically symmetrical for a co-falling observer, electromagnetic radiation is neither generated in the rest frame of the falling observer, nor in the rest frame of a far-away observer who is stationary with respect to the gravitating mass.

The undistorted spherically symmetrical shape of the electric field of the falling point charge can be subject to empirical testing. Since that shape of the electric field is required by the relativity principle, it is implicitly contained in Einstein's field equation (that is based on the relativity principle and on the invariance of the local speed of light). That is to say: If the electric field were distorted, the theory of General Relativity would be disproved.

b) In comparison, things are very different when an electric charge is "falling" in an external *electric* field (in the absence of gravity), i.e., if the charge is accelerated by the electric field between two capacitor plates. Then electromagnetic radiation is generated which is felt by a far-away observer, and for the co-falling observer the shape of the electrostatic field of the charge is distorted and no longer spherically symmetrical.[16,aa)]

This difference between the two scenarios provides justification for claiming that it is not only the charge, but also the surrounding space volume itself that, in case of a spherical symmetry of the electrostatic field of a charge falling in a gravity field, is in accelerated motion in the reference frame of a distant observer.

c) It was Einstein himself who, a few years prior to his death, mentioned the possibility of moving space volumes:[bb)]

> "When a smaller box is situated, relatively at rest, inside the hollow space of a larger box S, then the hollow space of s is a part of the hollow space of S, and the same 'space', which contains both of them, belongs to each of the boxes. When s is in motion with respect to S, however, the concept is less simple. One is then inclined to think that s encloses always the same space, but a variable part of the space S. It then becomes necessary to apportion to each box its particular space, not thought of as bounded, and to assume that these two spaces are in motion with respect to each other.
>
> Before one has become aware of this complication, space appears as an unbounded medium or container in which material objects swim around. But it must now be remembered that that there is an infinite number of spaces, which are in motion with respect to each other. The concept of space as something existing objectively and independent of things belongs to pre- scientific thought, but not so the idea of the existence of an infinite number of spaces in motion relatively to each other. This latter idea is indeed logically unavoidable, but is far from having played a considerable role even in scientific thought."

The importance of this Appendix has recently been stressed by C. Rovelli.[17,cc)]

E. Gravitational waves as an affirmation of the recognition that gravitational fields hold no energy

a) The existence of gravitational waves can be derived in two different ways. The first (and common) one is the following: The spherical gravitating mass shall no longer be at rest, but shall be in oscillating, accelerated motion. As a slight modification of the constellation that forms the basis of the Schwarzschild solution, not only the T^{00}-component, but also other components of T are supposed to be different from zero (though only slightly). As a consequence, the Schwarzschild metric, which, in the limit of large r, converts into the Minkowski metric of flat spacetime, is no longer exactly correct. Instead, the new metric tensor has components which differ from those of the Minkowski metric by small amounts. As a next step, one can show (by an approximation) that these deviations from the Minkowski metric are wavelike.

A second (and much easier) way to prove the existence of gravitational waves is the following: Eq. (9), in which gravitational acceleration is a function of r, presents itself as a perfect analogy to Coulomb's law of the electrostatic field. Consequently, E. M. Purcell's[dd)] derivation of an electric wave generated by an electric charge that starts or stops, which does not need a magnetic field but is getting by on the assumption (postulated by Special Relativity) that any change in the electric field propagates with velocity c, is valid also for a *gravitational* wave, provided one acknowledges that any change in the *gravitational* field, too, propagates with velocity c.

Purcell's derivation goes like this: He imagines an electric point charge that had been sitting at the origin of Cartesian coordinates for a long time. Eventually, at time zero, the charge accelerates abruptly over an infinitesimal short distance in the positive x-direction, and travels at constant velocity v along the positive x-axis thereafter. Purcell then draws a diagram that represents the situation at $t = 2$ units, when, for instance, two seconds have elapsed. Given that any change in the field propagates at velocity c but not faster, two regions in

[z)] For an illustration of the relativity principle as applied to a space ship coasting in free space—as another example of an inertial system—see R. P. Feynman, Lectures on Physics, Vol. 1, 1963, Chapter 15-4, page15-6: "The biologists and medical men sometimes say it is not quite certain that the time it takes for a cancer to develop will be longer in a spaceship, but from the viewpoint of a modern physicist it is nearly certain; otherwise one could use the rate of cancer development to determine the speed of the ship!"

[aa)] For a picture of the electric field lines of a formerly stationary point charge that has been accelerated, see E. M. Purcell, *Electricity and Magnetism*, 2nd ed. (1985), Chapter 5.7, Fig. 5.17, p. 189.

[bb)] A. Einstein, *Relativity – The Special and the General Theory* (Bonanza Books, 1961), Appendix V—supplemented in 1952 by Einstein—pp. 138 and 139.

[cc)] The Order of Time, 2017, p. 67, footnote.

[dd)] *Electricity and Magnetism*, 2nd ed. (McGraw-Hill Book Company, 1985), Chapter 5.7, pp. 187–191, and Appendix B, pp. 459–463.

the diagram have to be distinguished from each other: The first region is the one beyond a circle with a radius of two light seconds, drawn around the origin of coordinates. In this region, the electric field must still be exactly as it had been prior to $t = 0$; this is because the information that the charge is no longer at rest cannot have reached this region yet. The second region is the region within the circle. Here the "news" that the charge is in motion has reached every point of the region. In other words: The electric field in that region is the same as it would be at $t = 2$ (not: at $t = 0$) in case the charge had been travelling at constant velocity v for a long time already, However, this appears to result in the generation of loose ends of the field lines at the arc of the circle.

In the next step of Purcell's derivation of an electric wave, Gauss's law plays a crucial role: Since, due to the absence of any charge outside of the field-generating point charge, the electric field lines in the two regions cannot, according to Gauss's law, display any disruptions or gaps, there must be connections between them along the arc of the circle.

Moreover, since two field lines cannot intersect or coincide, the emerged field lines along the arc of the circle must be densely packed. It is in this region along the arc of the circle where the density of the electric field lines is much stronger than elsewhere.

Thereby a transversal electric wave has come into existence. But, to put it conversely: Without the zero divergence of the electric field in that special region as required by Gauss's law, there would be no electric wave.

The same is true for a gravitational wave that obeys analogous laws. Gauss's law—as a consequence of which the divergence of the electric field in vacuum is vanishing—is replaced by the recognition that there is a zero-divergence of the gravitational field in places where the Ricci tensor is zero, that is, in vacuum. Thereby the existence of a gravitational wavefront—with the gravitational field lines in that front oriented at right angle to the direction of propagation of the wavefront—is accounted for.

Given the divergence of the gravitational field is zero in the region of the gravitational wave, the gravitational field lines that constitute the wave front cannot be linked to any mass, and hence cannot be linked to any energy.

On top of this, the described phenomenon of a flow of space is thus made evident. This is because it is not the parallel gravitational field lines of the wave front that can be measured by instruments, but the deformation of space that goes along with the propagation of these field lines: Though the Ricci tensor is zero in the region of the wave, the Riemann tensor is not. This results in a temporary increase in distance between two fixed points in one direction (that direction being perpendicular to the direction of propagation of the wave), and, at the same time, to a temporary decrease in distance between two fixed points in a second direction (that, too, is perpendicular to the direction of propagation of the wave). But this is just another way of saying that elements of space volume are in motion. Note that these forces do not act in the direction of the gravitational field lines, and cannot, for this reason already, be true gravitational forces. Since a spring that connects said points would be subject to tension or to compression, it is made obvious that this motion of space is an accelerated one. The tidal-like forces that are observed when a gravitational wave hits matter are thus transformed away.

b) In case the wave is doing work on bodies, it is the hidden reservoir of energy that is tapped, and not the energy of the gravitational wave (which is zero). The common opinion on the energy of gravitational waves is contradictory in the following respect: If the energy density of the gravitational field is numerically negative (as is asserted by most authors), the energy contents of a volume of flat space will decrease—and not increase—when a gravitational wave is passing by. In order to save the principle of conservation of energy under the presumption that there is only the energy of the gravitational fields (of the wave and of the source bodies) and the kinetic energy of the source bodies, the two partners of a binary star system that, by means of fast rotation, shall be the source of the gravitational wave, should increase—and not decrease—the sum of their kinetic and potential energies as a result of the emission of gravitational waves, which would then lead to a higher orbit. But what we are told is just the opposite: The sum of the kinetic energies and the potential energies of the two partners of the binary star system is said to be *diminished* by the emission of gravitational waves, resulting in a diminishing of the radius of the orbit.

F. The indispensability of "dark energy" in General Relativity

a) If it is taken for granted that the sum of kinetic and potential energy of a system of gravitating bodies is constant over time, and if it is also taken for granted that the gravitational field cannot have or give off any energy, it follows that an increase in the total energy of a system goes along with an appropriate decrease in some other form of energy (and not in energy of a gravitational field). One should keep in mind that the seat of potential energy of a test body is never in the test body itself; when the potential is realized, energy disappears elsewhere, e.g., in the interior of some field.

In our special case, however, one should note that the other form of energy which is involved here comes into play only when the falling test body hits the surface of the gravitating mass, so that thermal energy is generated. As long as the test body is in free fall, its kinetic energy—and hence its mass—is the same as it was when the test body was at rest outside of the gravity field, provided one is using floating coordinates in the vicinity of a gravitating mass. In other words: The mass of freely falling bodies does not increase, quite different from the mass of a body that is gathering speed in flat spacetime. The absence of any relativistic mass of a body falling in a gravitational field has been acknowledged by textbooks, but it has not been recognized that the reason for this absence lies in the fact that space itself is in accelerated motion near a gravitating mass.If, in contrast, coordinates are used that do not participate in the flow of space near massive objects (but which are immovable in the frame of reference used), the lack of an increase in true kinetic energy of the falling body is hidden from view, as the freely falling body is gathering speed and is therefore increasing its apparent kinetic energy in that rigid frame of reference.

No matter which of the two types of coordinates are used, General Relativity thus cannot dispense of a "dark energy reserve" for the description of everyday phenomena in nature (including Newton's apple). Given that the gravity field holds no energy, the second term in Eq. (2) cannot, as Einstein asserted, be an expression of the energy of the gravitational field (that is being converted into ordinary energy or vice versa), but must be an expression of the energy of a hidden and thus "dark" reservoir of energy.

The seat of that dark energy cannot be found in three-dimensional space. Otherwise that energy would contribute to the curvature of space and hence to the intensity of the gravitational force. Instead, one cannot do without the assumption of a fourth spatial dimension. It is in that fourth direction where the bulk of all dark energy must be located.

Thereby Schrödinger's vision (cited above) of "new foundations" of the energy conservation principle—that are required as soon as one is realizing that the gravity field is void of energy—is coming true.

b) Of course, in order to solve the task of quantifying the amount of energy absorbed or given off by the reserve of dark energy, one is free to act as if the gravity field were in possession of a nonvanishing energy whose density is proportional to the negative square of the gravitational intensity in space. Nevertheless, one should always be aware of the fact that during a Helmholtz-contraction of a hollow gravitating sphere (by which, in theory, an infinite amount of mechanical work can be gained according to Newtonian physics), the field-free (!) space in the interior does not, contrary to common belief, possess a gravitational field with an infinite energy density. At that location, objects are rather in communication with an inexhaustible reserve of dark energy.

c) The cosmological constant λ, which has been correctly regarded as an expression of dark energy, stands for nothing but that small fraction of dark energy which is *not* located in the direction of a fourth spatial dimension.

G. The indispensability of dark energy in the limit of Newtonian gravity

As regards the gravitational acceleration as a function of r, there is no difference between Newtonian physics and General Relativity [see Eq. (9)]. It is the difference between r and R, and the difference between t and τ that vanishes in the Newtonian limit.

Both according to the Schwarzschild metric and according to Newton's law of gravitation, any closed-loop integral of $g\ ds$ is zero. Of course, the straight-path integral of $g\ ds$ along a radial distance is *not* zero. In Newtonian physics, the amount of work gained on such a path (and expressed by that integral) is said to be exactly compensated by a loss in "potential energy." But this does not convey any more information than the phrase "any closed-loop integral of $g\ ds$ is zero" does. Instead, it is just another way of saying the same thing.

In the 19th century, the notion of a "gravitational field" was introduced into Newtonian physics. But Newton's law of gravitation did not disclose how the energy density of that field should be quantified. Analogous to the electric or magnetic field, the energy density was supposed to be proportional to the numerically positive square of the field intensity g by some authors (for instance, by J. C. Maxwell and O. Heaviside), but this led to unsurmountable difficulties, especially to a conflict with the principle of energy conservation, unless a hidden energy reservoir would be introduced: If two mutually attractive gravitating bodies move toward each other, mechanical work is yielded, but not at the expense of the numerically positive energy of the gravitational field. Instead, the energy of the gravitational field is increased (!) due to the postulated quadratic relationship between the energy density of the field and its intensity. As O. Heaviside put it:[18,ee)]

> "Now there is a magnetic problem in which we have a kind of similarity of behavior, viz., when currents in material circuits are allowed to attract one another. Let, for completeness, the initial state be one of infinitely wide separation of infinitely small filamentary currents in closed circuits. Then, on concentration to any other state, the work done by the attractive forces is represented by the sum of $\mu H^2/2$, where μ is the inductivity and H the magnetic force. This has its equivalent in the energy of motion of the circuits, or may be imagined to be so converted, or else wasted by friction, if we like. But, over and above this energy, the same amount, the sum of $\mu H^2/2$, represents the energy of the magnetic field, which can be got out of it in work. It was zero at the beginning. Now, as Lord Kelvin showed, this double work is accounted for by extra work in the batteries or other sources required to maintain the currents constant. (I have omitted reference to the waste of energy due to electrical resistance, to avoid complications.) In the gravitational case there is a partial analogy, but the matter is all along assumed to be incapable of variation, and not to require any supply of energy to keep it constant. If we asserted that $ce^2/2$ was stored energy [e is the intensity of the gravitational field, c is a positive constant], then its double would be the work done per unit volume by letting bodies attract from infinity, without any apparent source."

Most authors therefore (or for other reasons) believed that the energy density of the gravitational field had to be proportional to the numerically *negative* square of the field intensity g. But then a contraction of a spherical shell built of ordinary matter (Kelvin–Helmholtz-contraction) would, according to Newton's law of gravitation, be able to yield an infinite amount of energy by harnessing the energy of a field-free volume of space in the interior of the hollow sphere. This was indistinguishable from the postulate of a hidden reserve of energy in field-free space equipped with an infinite energy density.

[ee)] O. Heaviside, "A gravitational and electromagnetic analogy," The Electrician **31**, 281 and 282 (1893), also found in O. Heaviside, *Electromagnetic-Theory*, Vol. 1, London 1898, pp. 461/462.

If, on the basis of Newton's law of gravitation, one would have asserted (as a third option) that the gravitational field holds no energy at all, the existence of a hidden energy reservoir would have been indispensable in an obvious manner right from the start.

Hence, no matter which of the three options regarding the energy density of the gravitational field would have been chosen, there would have been no way to avoid the postulate of a hidden reservoir of energy in space on the basis of Newton's law of gravitation.

H. Means of empirically corroborating the hypothesis of a hidden energy reservoir in the fourth spatial dimension

The hypothesis of a harnessing of a hidden energy reservoir in the simple case of Newton's apple may undergo an empirical test. This test would consist in checking whether Eq. (9), which is identical with Newton's law of gravitation (if r appearing in that law stands for circumference of a circle around the center of mass divided by 2π, and not for the number of radially oriented meter sticks laid end to end), is empirically correct for local observers even in the vicinity of very heavy masses. If so, proof would thereby be obtained for the postulate (shared by Wheeler, Thorne, and Misner) that the gravitational field has no mass.

It should be noted that the general attitude (based on Einstein's remark cited above) is quite to the contrary.[ff)]

As the equivalence of mass and energy has been empirically corroborated already, it would thereby, i.e., by the same token, be empirically proved that the gravitational field around a spherical mass holds no energy.

In the face of the undisputed empirical fact that Newton's apple generates thermal energy when coming to rest on the ground, the existence of a reservoir of energy hidden in a fourth spatial dimension would thereby be indirectly proved, given that the energy which has turned up as thermal energy cannot come from the gravitational field, and given that the principle of local conservation of energy is correct.

It is worthwhile noticing that in recent years the existence of a hidden energy sink in the fourth spatial dimension was attempted to be proved empirically in the Fermi National Accelerator Laboratory (by a team led by G. Landsberg of Brown University), and in DESY's electron-proton collider in Germany, *in a closely analogous manner*. See K. Tuttle:[19,gg)]

> "Collision experiments carefully reconstruct all particles emerging from a collision. A possible sign of extra dimensions would be a collision in which a particle—and hence energy—'disappeared,' perhaps indicating a graviton leaving our visible universe and entering extra spatial dimensions—the megaverse."

The existence of a hidden energy sink in the fourth spatial dimension would thus have been proved indirectly but compellingly by an observation which would have constituted an apparent violation of the principle of energy conservation. A somewhat easier—though similar—way to prove the existence of a hidden and thus "dark" energy reservoir in the fourth spatial dimension consists—as has been described above—in the testing of Newton's law in the vicinity of very heavy masses (with r that appears in Newton's law standing for circumference of a circle around the center of mass divided by 2π, and not for the number of radially oriented meter sticks laid end to end). A positive outcome will constitute an apparent violation of the principle of energy conservation, which will—in order to avoid such a violation—compel us to accept a fourth spatial dimension, quite similar to the test scenario of the collision experiments. This empirical check of Newton's law near very heavy masses would also represent another test of General Relativity, since, in the case of a negative outcome, both Newton's law and General Relativity (whose predictions do not differ from each other insofar) would be disproved.

But are we really in need of such an empirical corroboration? When accepting the fact that the existence of a hidden energy reservoir in the fourth spatial dimension is a mathematical consequence of Einstein's field equation and of the principle of local energy conservation, and given the fact that these two principles have been successfully tested in numerous experiments already, any further empirical test can hardly be said to be indispensable.

I. The nature of weight felt by stationary objects in gravity fields

a) For a distant observer who uses a rigid coordinate system (frame of reference), a test body which is falling at (r-dependent) escape velocity in a gravity field of a heavy mass is hence stationary relative to a flowing space volume element that surrounds the test body (while the test body is *not* stationary relative to the rigid system of coordinates the distant observer is using).

A test body freely falling at a velocity less than the r-dependent escape velocity does not change its velocity relative to the moving space volume element by which it is surrounded at any moment in time.

A test body sitting on the surface of the gravitating, spherical mass is not experiencing a downward force of gravity, but is experiencing upward intermolecular, repulsive forces from the surface on which it is sitting. These upward, intermolecular forces (that are counteracted by the inertia of the test body) do not manage, though, to set the test body in motion with respect to the surface of

[ff)]As an example, see the entry "vacuum solutions" in the English Wikipedia: "… this gravitational field energy itself produces more gravity. This means that the gravitational field outside the Sun is a bit stronger according to general relativity than it is according to Newton's theory."

[gg)]"The Search for Extra Dimensions," Symmetry Magazine, published by Fermilab/SLAC, Vol. II, Issue 10, December 2005/January 2006. See also H. Reichenbach, op. cit., p. 275/281/282: "… the principle of action by contact: causal effects cannot reach distant points of space without having previously passed through intermediate points. … This rule determines the dimensionality of space … Let us assume that the three dimensions of space are visualized in the customary fashion, and let us substitute a color for the fourth dimension. … The fact that a closed three-dimensional surface no longer encloses a spatial region will become clear from the following consideration. If we lock a number of flies into a red glass globe, they may yet escape: they may change their color to blue and are then able to penetrate the red globe."

the spherical mass, but they do manage to set the test body in accelerated motion relative to a space volume element that is passing by.

The situation is analogous to an observer in a capsule hundreds of millions of light years away that is connected to planet Earth (and the Milky Way) by a long tether. Due to the expansion of space, the tether is under permanent mechanical tension, and the observer in the capsule feels a force that he might interpret as gravity [his time τ is described in Eq. (12)]. The observer would be wrong, though: It is not *gravity* that is acting on the capsule and its contents, but the force exerted by the tether.

IV. DISCUSSION

But what if the test body is radially *rising* in free motion in the gravity field at (r-dependent) escape velocity? In that case, gravity in the reference frame of the distant observer can only be transformed away by assuming the existence of space volume elements that do not flow *toward* the gravitating mass, but *move away* from this mass (thereby reducing their velocity, which is always equal to the r-dependent escape velocity).

How can space flow toward the gravitating mass and also move away from it? One should recall that Einstein mentioned (in 1952) the possibility of numerous, different velocities of space volumes as "logically unavoidable," without, however, providing physically relevant examples. If one assumes that space volume elements are permanently flowing toward the gravitating mass even when there is no test body in the gravity field, one is compelled to rate the free *rise* of space volume elements as being a result of a partial time reversal.

There is no other way of transforming the gravitational force away. The transforming away of the gravitational force, in turn, is indispensable for explaining how the mass that is freely rising in a gravitational field is being decelerated without transferring energy to the gravitational field (which is impossible due to its zero energy density) or to the energy reservoir of some dark, decelerating force (which would, in conflict with the relativity principle, lead—in the reference frame of the rising mass—to a deformation of the spherically symmetrical shape of the electric field the rising mass shall be imagined to be in possession of).

A partial time reversal, that is, the encounter of the "big arrow of time" with a much smaller one, is known in principle. The existence of antimatter and also of hole conduction in semiconductors can be described in this fashion. However, so far only phenomena that occur on the *microscopic* scale have been candidates for representing "small arrows of time." This limitation is no longer justified. As a consequence, the concept of a definite, uniform direction of time as something intrinsic to the macroscopic objects is rigorously destroyed.

Since gravitation is not a true force, and since the gravitational field has no energy (quite different from an electric or magnetic field), it is hardly conceivable that, on the level of quantum mechanics, gravitation is put into effect by particles, that is by gravitons.

[1]A. Einstein, *The Meaning of Relativity*, 5th ed. (Princeton U.P., Princeton, NJ, 1956).
[2]A. Einstein, *The Foundation of the General Theory of Relativity*, translated from "Die Grundlage der allgemeinen Relativitätstheorie," Ann. Phys. **354**, 769 (1916), p. 769, in: A. Einstein, H. A. Lorentz, and H. Minkowski, *The Principle of Relativity* (Dover Publ., Mineola, NY, 1952).
[3]W. Pauli, *Theory of Relativity* (Dover Publ., Mineola, NY, 1981).
[4]A. Einstein, "Sitzungsberichte der königlich preußischen Akademie der Wissenschaften," Semi **I**, 448 (1918).
[5]H. Bauer, Phys. Z. **19**, 163 (1918).
[6]B. Greene, *The Hidden Reality, Parallel Universes and the Deep Laws of the Cosmos* (Vintage Books, New York, 2011).
[7]A. Vilenkin, *Many Worlds in One—The Search for Other Universes* (Hill and Wang, a division of Farrar, Straus, and Giroux, New York, 2006).
[8]A. Einstein, "Sitzungsberichte der königlich preußischen Akademie der Wissenschaften," Semi **1**, 154 (1918).
[9]A. Einstein, Phys. Z. **19**, 115 (1918).
[10]J. Droste, Proc. R. Netherlands Acad. Sci. **19I**, 197 (1917).
[11]C. M. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, CA, 1973).
[12]E. Schrödinger, Phys. Z. **19**, 4 (1918).
[13]A. Einstein, *Relativity—The Special and the General Theory* (Bonanza Books, Mineola, NY, 1961).
[14]R. P. Feynman, *Lectures on Physics* (Addison-Wesley, Boston, MA, 1963), Vol. 1.
[15]H. Reichenbach, *The Philosophy of Space and Time* (Dover Publ., Mineola, NY, 1958).
[16]E. M. Purcell, *Electricity and Magnetism*, 2nd ed. (McGraw-Hill Book Company, New York, 1985).
[17]C. Rovelli, *The Order of Time* (Riverhead Books, New York, 2017).
[18]O. Heaviside, *Electrician* 31, 281 (1893); *Electromagnetic Theory* (The Electrician, London, 1898), Vol. 1.
[19]K. Tuttle, Symmetry Mag. **II**(10) (2006).