The energy density of the gravitational field

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Abstract: It is shown that 100-year-old, conflicting ideas on the positive or negative energy of the gravity field collide with the principle of local conservation of energy. A scrutiny of the Schwarzschild metric, carried out with a different method than that applied by E. Schrödinger but completed with a similar result, re-confirms that the gravity field holds no energy at all, with that recognition being tacitly acknowledged by Misner, Thorne and Wheeler in 1973. Given that it does not hold any energy, it cannot, by definition, be qualified as a force-field. Given that it is not a force-field, it is capable of being completely transformed away even in the rigid reference-frame of a distant observer outside of the field. Contrary to what (early) Einstein believed, this can (and must) be achieved by the concept of "flowing spaces" that was introduced by elder Einstein himself in 1952. It is shown that this concept leads to empirical consequences. Moreover, the energy of the gravity field is necessarily replaced by an inexhaustible "dark energy", which flows into any massive object (including Newton's apple) whenever, after a free fall, it is being decelerated. Thereby Schrödinger's vision of "new foundations" of the energy conservation principle (as a consequence of his recognition that the gravity field holds no energy) is coming true. Because of the absence of any gravitational field lines that originate from that energy, the (main) seat of this "dark energy" cannot be in three-dimensional space, but must sit at a location separated from ordinary space by a short distance in a direction perpendicular to all three ordinary spatial directions.

Keywords: Energy of the gravitational field; General Relativity; dark energy; fourth spatial dimension; time reversal

I) Introduction

Force fields are characterized by their property of being capable of transferring energy to a body they act on, at the expense of energy previously stored in that field.

The electric field serves as a perfect example. An electrically charged sphere that finds itself between the plates of a charged parallel-plate-capacitor is accelerated by the field and is thereby picking up kinetic energy. During this process, the Poynting-vector field **E x B** shows a flow of energy from the field that surrounds the sphere into that sphere. The sphere is a sink, and the space around the sphere is a source of that energy flow.

Moreover, since the electric field between the capacitor plates holds energy, it has mass.

Suppose we would find that, contrary to our assumption, the electric field between the two capacitor plates would have no (inert and heavy) mass. On the basis of the principle of equivalence of mass and energy, we would have to come to the conclusion that it holds no energy. Moreover, given it holds no energy, it cannot be a force field. For in order to be a force field, it would, by definition, have to be in possession of energy. That energy is transferred to the sphere, where it is converted into kinetic energy. Without any energy contents, the electric field would be the field of a pseudo force, and the kinetic energy of the sphere would have to be the result of a tapping of a hidden energy reservoir

Let us now switch from the electric to the gravitational field. Unfortunately, General Relativity does not provide a tool analogous to the Poynting vector that would make the flow of energy visible that enters Newton's apple when it is falling from the tree. Even worse, there is no consensus on how the energy density of the gravitational field can be quantified. An analogy with the electric field, according to which the energy density would be proportional to the strength of the local gravitational field (force per kg) squared, and would thus have a positive numerical value, leads to an apparent conflict with the principle of conservation of energy, and is therefore dismissed by modern textbooks. Though some authors postulate the energy density of the gravitational field as being proportional to the negative square of the strength of the gravitational field, this postulate was rejected by Einstein for good reasons.

In this article, the energy density of the gravitational field (if there is any) shall be determined once and for all.

II) Methods

No other methods than logical and mathematical conclusions are applied. The relativity principle, Einstein's field equation, the Schwarzschild solution of Einstein's field equation, the equivalence of mass and energy (as postulated by Special Relativity), the equation of a geodesic, and the local principle of conservation of energy are used as starting-points.

III) Results

1) The quest for a determination of the energy density of the gravity field

a) Einstein's field equation of General Relativity (see its formulation by A. Einstein, "The Meaning of Relativity", Princeton University Press, 5th edition 1956, therein: "The General Theory", p. 84, Equation 96), that is,
(1)

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G \; T^{\mu\nu}$$

presupposes the vanishing of the covariant divergence of the "ordinary" energy-momentum tensor **T** that comprises energy and momentum of ordinary matter **M** (but not of matter or momentum ascribed to a gravitational field). **R** denotes the Ricci-Tensor, **g** denotes the metric tensor, **G** denotes Newton's gravitational constant. The indices **μ** und **nu** run from 0 to 3, and represent the four components (one is temporal and three are spatial), that is $\mathbf{x^0}$, $\mathbf{x^1}$, $\mathbf{x^2}$, $\mathbf{x^3}$, of a four-dimensional vector (of which there are four associated with each 4 x 4 tensor).

Proof: The covariant divergence of the left-hand side of the equation is zero as a mathematical necessity (a detailed proof is omitted here). Consequently, the covariant divergence of the right-hand side of the equation, too, must vanish.

The vanishing of the ordinary divergence of the energy-momentum-tensor **T** seems to be an expression of the principle of conservation of energy. In order to visualize this, we choose a reference frame in which no mass is changing its spatial position over time. Consequently, the energy-momentum-tensor **T** has one single component only, that is, a component in the $\mathbf{x}^0$ (temporal)- direction. Given that the mass is spread out in space, the energy-momentum tensor field (in that reference frame) is a vector field whose direction is parallel to the $\mathbf{x}^0$-axis, and whose magnitude is that of the local mass density. If that vector-field had an ordinary divergence different from zero, the principle of conservation of energy(=mass) would seemingly be violated, as mass (=energy) would appear or disappear into nothingness.

But what about the energy and mass ascribed to the gravitational field? As A. Einstein put it ("The Meaning of Relativity", Princeton University Press, 5th edition 1956, therein: The General Theory, p. 83): *"It must be remembered that besides the energy density of the matter there must also be given an energy density of the gravitational field, so that there can be no talk of principles of conservation of energy and momentum alone.*"

As a consequence, Einstein postulated the following equation as an expression of the principle of conservation of energy ("The foundation of the general theory of relativity", translated from "Die Grundlage der allgemeinen Relativitätstheorie", Annalen der Physik, Vol. 354 – 1916 – , pp. 769, in: A. Einstein/H.A. Lorentz/ H. Minkowski/,"The Principle of Relativity", Dover Publ. 1952, § 18, Equation 57 and 57a, p. 151):
(2)

$$0 = \frac{\delta T^{\alpha}_{\sigma}}{\delta x_{\alpha}} + \Gamma_{\sigma\beta}\, T^{\alpha}_{\beta} = \frac{\delta T^{\alpha}_{\sigma}}{\delta x_{\alpha}} + \frac{1}{2}\,\frac{\delta g^{\mu\nu}}{\delta x_{\sigma}}\, T_{\mu\nu}$$

This equation (2) is in accordance with (1), as it states that the covariant divergence of **T** (not the ordinary one) is zero, as is mathematically presupposed by (1). The covariant divergence is distinguished from the ordinary divergence by a summand that contains the Christoffel symbol.

One is getting a deeper understanding of (2) by imagining a test body that, from the perspective of a far-away observer, is "gathering speed" in a gravity field. In such a case, **T** is of a kind that makes (2) an expression of a geodesic along which the test body is coasting when left to itself (W. Pauli, Theory of Relativity, Dover Publ. 1981, Section 54, p. 158). If the gradient of the components of the metric tensor **g** in (2) is not vanishing, the ordinary divergence of **T** is different from zero. The non-vanishing, ordinary divergence of **T** is compensated by the last summand on the very right side of (2). In Einstein's view, kinetic energy of the falling body is thus converted into energy of the gravitational field, or vice versa (see A. Einstein, op. cit, after Equation 57):

"*Physically, the occurrence of the second term on the left-hand side* [this term of Einstein's equals the last term on the very right side of our Equation 2] *shows that laws of conservation* *of momentum and energy do not apply in the strict sense for matter alone, or else that they apply only when the g are constant, i.e. when the field intensities of gravitation vanish. This second term is an expression for momentum, and for energy, as transferred per unit of volume and time from the gravitational field to matter.*"

From (2), one can derive (see A. Einstein, "The foundation of the general theory of relativity", § 17, Equation 56, p. 150; A. Einstein, "*Der Energiesatz in der allgemeinen Relativitätstheorie*", Sitzungsberichte der Preußischen Akademie der Wissenschaften, 1918, Vol. 1, pp. 448-459):
(3)

$$0 = \frac{\delta(T_\mu^\sigma + t_\mu^\sigma)}{\delta x_\sigma}$$

According to Einstein, **t** is an expression of the density of "momenergy" ascribed to the gravitational field. Einstein thus asserts that the ordinary divergence of **T** plus the ordinary divergence of **t** amount to zero.

In the simple case in which (by arrangement) the only non-vanishing component of **T** inside a material body is $\mathbf{T_0^0}$ (mass at permanent rest), and in which all components of **T** outside the mass are zero, the ordinary divergence of **T** is zero. Then, however, the ordinary divergence of **t**, too, must be zero according to (3).

But does this allow the determination of the components of **t**, especially of the magnitude of $\mathbf{t_0^0}$? The answer is in the negative.

b) After all, an integral which is founded on (3), that is (here the fourth component is the time-like component)
(4)

$$\int_V (T_i^4 + t_i^4) dx^1 dx^2 dx^3 = const$$

has (under certain restrictions) the same value in any reference frame (see W. Pauli, Theory of Relativity, Dover Publ. 1981, Section 61, Equation 447, p. 176; A. Einstein, "*Der Energiesatz in der allgemeinen Relativitätstheorie*", Sitzungsberichte der Preußischen Akademie der Wissenschaften, 1918, Vol. 1, pp. 448-459, Equation. 25). That is to say: The sum of all "momenergies" of bodies and gravitational fields is not only constant over time, but is also the same in any frame of reference. For comparison, in Special Relativity the "momenergy" of a body has the same magnitude in any frame of reference, and is equal to the mass of the body in its own rest frame. In General Relativity, it is the sum of the "momenergies" of bodies and gravitational fields that apparently replaces the "momenergies" of bodies alone.

Given the indeterminateness of the magnitude of the components of **t**, it is no longer a surprise to find that the postulated energy-momentum density **t** of the gravitational field, and hence also its ordinary energy density $\mathbf{t_0^0}$, can be made to vanish by means of an appropriate choice of "flat" coordinates on the basis of (3) [see W. Pauli, Theory of Relativity, Dover Publ. 1981, Section 61 , p. 176: "*Since these quantities do not depend on the derivatives of the g higher than the first, we can conclude immediately that they can be made to vanish at an arbitrarily prescribed world point for a suitable choice of the coordinate system (geodesic reference system).*"].

Even more: One- and the same observer arrives at completely different results for **t** and hence for the energy density $\mathbf{t_0^0}$ of the gravitational field, depending on which coordinate system he is using (see H. Bauer, "Über die Energiekomponenten des Gravitationsfeldes", Physikalische Zeitschrift, Vol. 19 – 1918 – , pp.163-165: "*As a concluding remark, we may state that the 'energy components' t have nothing to do with the existence of a gravitational field, but depend on the choice of coordinates only...*").

In the face of this recognition, W. Pauli drew the following conclusion (W. Pauli, Theory of Relativity, Dover Publ. 1981, Section 61, p. 177):

"*According to this, one cannot asign any physical meaning to the values of the t themselves, i.e. it is impossible to carry out a localization of energy and momentum in a gravitational field in a generally covariant and physically satisfactory way.* "

Hence, no definite, time- and space-dependent value can be attributed to the local energy density of the gravitational field (at least not on the basis of Equation 3).

c) This stance of Einstein's and Pauli's presents a sharp contrast to the long-standing and still popular assertion according to which the energy density of the gravitational field is proportional to the negative square of the local intensity of the gravitational field. That assertion has been backed by the following argument: Since General Relativity can be expected to be almost indistinguishable from Newton's physics when it comes to objects like the solar system, and since Newtonian physics has been understood as equalling the negative potential energy and the energy of the gravitational field, one might be inclined to assume that the energy density near gravitating masses has to be negative also in General Relativity. See *W. Pauli* (Theory of Relativity, Dover Publ. 1981, Section 61, p. 176):

"*In the earlier field theories of gravitation already, it was the sign of the energy density of the gravitational field which had led to difficulties. In spite of these difficulties it would, on physical gounds, be hard to abandon the requirement that an analogue to the energy- and momentum-integrals of Newtonian theory should exist.*"

This led *T. Levi-Civita* and *H.A. Lorentz* to the belief that the sum of positive energy of matter and (postulated) negative energy of the gravitational field is always zero in a closed system. Even today, this belief is very popular among physicsts [see only Brian Greene, "The Hidden Reality, Parallel Universes and the Deep Laws of the Cosmos", 2011, Note 9 to pages 65-70 – Chapter 3 – , p. 381: "*The gravitational field can supply the particles with such positive energy because gravity can draw down its own energy reserve, which becomes arbitrarily negative in the process: the closer the particles approach each other, the more* *negative the gravitational energy becomes (equivalently, the more positive the energy you'd need to inject to overcome the force of gravity and separate the particles once again). Gravity is thus like a bank that has a bottomless credit line and so can lend endless amounts of money; the gravitational field can supply endless amounts of energy because its own energy can become ever more negative. And that's the energy source that inflationary expansion taps*."; see also Alex Vilenkin, "Many Worlds in One – The Search for Other Universes", 2006, Part I 1, pp. 11/12: "*So the energy of the inflating chunk must also have grown by a collossal factor, while energy conservation requires that it should remain constant. The paradox disappears if one remembers to include the contribution to the energy due to gravity. It has long been known that gravitational energy is always negative. This fact did not appear very important, but now it suddenly aquired a cosmic significance. As the positive energy of matter grows, it is balanced by the growing negative gravitational energy. The total energy remains constant, as demanded by the conservation law*."].

Einstein rejected this assertion categorically [see his paper: "Über Gravitationswellen", Sitzungsberichte der königlich preußischen Akademie der Wissenschaften, 1918, Semi-Volume 1, pp. 154-167 [167]], because it would then be possible "*for a material system to vanish into nothing without leaving a trace*."

Moreover, one cannot but acknowledge that **t** is spatially and temporally indeterminate (on the basis of Equation 3). On the basis of (3), it is nothing but an arbitrary stipulation to assert that the energy density of the gravitational field is proportional to the negative square of its magnitude.

d) Despite the impossibility of giving the energy density and the momentum density of the gravity field a definite value at a certain place and time, equation (4) is meaningful to Pauli and Einstein, since that equation appears to make it possible to (see Pauli, Theory of Relativity, Dover Publ. 1981, Section 61, p. 177) "*calculate the change in the material energy of a closed system in a simple fashion*.". For Pauli and Einstein, the principle of conservation of energy is observed in General Relativity by means of equation (4). Einstein put it the following way (A. Einstein, "Der Energiesatz in der allgemeinen Relativitätstheorie", op. cit., p. 452):

"*Contrary to our present thinking habits, we thus arrive at attributing more reality value to an integral than to its differentials*."

But such a reasoning can be convincing only in case the principle of conservation of energy (and also the principle of conservation of momentum) is not understood as a *local* principle. Otherwise, that is, if one does understand the energy principle as a local one (saying that energy can never simply appear or disappear at some location, but can only flow into that location or out of it), then the two equations (3) and (4) – given that the local energy density is indeterminate in any frame of reference – are not sufficient to guarantee the principle of energy conservation in General Relativity.

This insufficiency surfaces in an obvious manner in Einstein's short paper "Notiz zu E. Schrödingers Arbeit 'Die Energiekomponenten des Gravitationsfeldes' " (Physikalische Zeitschrift, Vol. 19 – 1918 – , pp. 115/116):

"*There can well be gravitational fields without tensions and without energy density*."

But when a test body is "gathering speed" in a gravitational field that is supposed to be a force field, energy has to flow into the test body from its immediate surroundings. This is similar to the case of an electric charge set into accelerated motion by an electric field, where the energy-flow (from the adjacent electric field into the accelerating charge) is made "visible" by the Poynting vector. Presuming (for a short while at least) that no energy reservoir other than the gravitational field is available, energy has to flow from the gravity field into the test body. Then, however, the gravitational field (assumed to be a force field) cannot have zero energy density at this location (contrary to Einstein's view).

To put it the other way around: If the gravity field is void of energy, it cannot be a force field. Otherwise the principle of local conservation of energy would be violated. It even makes perfect sense to define a force field as a field whose energy is transferred to an accelerated body. Hence, if a field has no energy, it cannot be a force field by definition.

2) The absence of any heavy mass (and hence of any energy density) of a gravitational field

a) If a gravity field were in possession of energy, it would have to have heavy mass [see A. Einstein, "The foundation of the general theory of relativity", translated from "Die Grundlage der allgemeinen Relativitätstheorie", Annalen der Physik, Vol. 354 – 1916 – , pp. 769, in: A. Einstein/H.A. Lorentz/ H. Minkowski,"The Principle of Relativity", Dover Publ. 1952, § 16, p. 148: "*For if we consider a complete system (e.g. the solar system), the total mass of the system, and therefore its total gravitating action as well, will depend on the total energy of the system, and therefore on the ponderable energy together with the gravitational energy*."].

One can, however, show that the gravity field does not have any heavy mass (or any energy). The absence of any energy of the gravity field follows from the Schwarzschild-equation (as a solution of Einstein's field equation for a non-rotating, spherical mass and its surroundings). According to the Schwarzschild-equation, the intensity of the gravitational "force" is inversely proportional to the square of the distance **r** (=circumference of a circle, divided by 2 pi) from the central spherical body, and directly proportional to its ordinary mass **M**, exactly the same as in Newton's law of gravitation.

Proof: If, in the formula for a **r**-geodesic, that is
(5)

$$\frac{d^2R}{d\tau^2} + \Gamma^1_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = \frac{d^2r}{d\tau^2} + \Gamma^r_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

$\mathbf{d^2x^1/dtau^2}$ is exchanged for $\mathbf{d^2r/dtau^2}$, and if the Christoffel symbol, that is
(6)

$$\Gamma^{p}_{\mu\nu} = \frac{g^{pn}}{2}\left(\frac{\delta g_{n\mu}}{\delta x^{\nu}} + \frac{\delta g_{n\nu}}{\delta x^{\mu}} - \frac{\delta g_{\mu\nu}}{\delta x^{n}}\right)$$

is written in full detail on the basis of the Schwarzschild metric, that is, on the basis of the tensor
(7)

$$g_{\mu\nu} = \begin{matrix} 1-\frac{2GM}{c^2 r} & 0 & 0 & 0 \\ 0 & -[c^2(1-\frac{2GM}{c^2 r})]^{-1} & 0 & 0 \\ 0 & 0 & -\frac{r^2}{c^2} & 0 \\ 0 & 0 & 0 & -\sin^2\theta \end{matrix}$$

we get:
(8)

$$\frac{d^2R}{d\tau^2}\frac{d\tau^2}{dt^2}\frac{1}{r(1 - r_s/r)} = -\frac{c^2 r_s}{2r^3} - \frac{r_s}{2(1-r_s/r)^2 r^3}\frac{dr^2}{dt^2} - \frac{d\varphi^2}{dt^2}$$

R stands for the radial distance measured by laying meter sticks end to end; **r** stands for the circumference of a circle around the spherical mass, divided by **2 pi**.

Since, according to the Schwarzschild metric, **dtau²/dt²** times $\mathbf{(1-r_s/r)^{-1}}$ equals unity if **tau** is the time of a (at least momentarily) stationary observer in the gravity field, and since with respect to a stationary object both **dr/dt** and **dphi/dt** are zero, we get for this situation from (8):
(9)

$$\frac{d^2R}{d\tau^2} = -\frac{c^2 r_s}{2r^2} = -\frac{MG}{r^2}$$

or:
(10)

$$\frac{d^2R}{dt^2} = -\frac{c^2 r_s}{2r^2}(1 - \frac{r_s}{r})$$

(10) is identical with the result obtained by Droste in 1917 (J. Droste, “The field of a single center in Einstein’s theory of gravitation and the motion of a particle in that field”, Proceedings of the Royal Netherlands Academy of Science, Vol 19 I, 1917, pp. 197-215, especially page 203). **M** is the ordinary mass of the spherical body, **G** is Newton’s

gravitational constant.

For a momentarily stationary observer in the gravity field, the gravitational acceleration **d²R/dtau²** , that is, the gravitational "force" per unit mass of a test body, is, according to (9), directly proportional to the mass **M** of the gravitating body (and does not differ from the acceleration yielded by Newton's law of gravitation). For if one doubles the ordinary density and hence the ordinary mass of the gravitating body, one doubles the gravitational "force".

If a heavy mass of the gravitational field appeared in the Schwarzschild-equation in a hidden manner and thus co-determined the intensity of the gravitational "force", and if one regarded the energy of the gravity field as being proportional to the positive or negative square of the intensity of the gravity field, a direct proportionality between the ordinary mass **M** and the gravitational "force" (as given by equation 9) would be unexplainable because of that quadratic relationship.

On top of this, (9) proves that the local gravitational "force", that is the "force" felt by a stationary observer in the field, is the same function of **r** as it is according to Newton's law of gravitation. Then the density of gravitional field lines, too, is the same expression of the local gravitational "force" as it is according to Newton's law of gravitation. Consequently, the gravitational field lines are divergenceless according to (9). This, too, shows: The gravity field that exists in empty space does not exhibit a heavy mass. Even more: There is no additional mass (=energy) at all – that would sit inside or outside of the central body – besides the ordinary gravitating mass.

Note that the absence of any mass or energy of the gravitational field occurs despite the fact that the components of the metric tensor **g** shown in (7) are not all zero or unity.

From this follows (recapitulated): According to the Schwarzschild-equation and hence according to General Relativity, heavy mass only exists in the form of ordinary mass **M**, not in the form of mass of the gravitational field. Hence, according to Special Relativity and its principle of equivalence of mass and energy, the gravitational field cannot have any energy.

Such a result is tacitly – though not explitly – acknowledged by C.W. Misner, K.S. Thorne, J.A. Wheeler in their famous standard textbook on gravitation (Gravitation, 1973, Chapter 20.4: Why the energy of the gravitational field cannot be localized, p. 467):

"*Not one of these properties does 'local gravitational energy-momentum' possess. There is no unique formula for it, but a multitude of quite distinct formulas. The two cited are only two among an infinity. Moreover, 'local gravitational energy-momentum' has no weight. It does not curve space. It does not serve as a source term on the righthand side of Einstein's field equations*."

With the local gravitational energy-momentum having no weight, and given the equivalence of heavy mass and energy, the gravitational field has no energy.

b) This recognition was anticipated by E. Schrödinger ("Die Energiekomponenten des Gravitationsfeldes", Physikalische Zeitschrift, Vol. 19 – 1918 –, pp. 4-7). Schrödinger combined (3) and the Schwarzschild-solution. He thereby achieved the following result (op. cit., pp. 6/7):

"*As mentioned above in an anticipating manner, it follows ... that **t** vanishes everywhere (outside of the gravitating sphere) identically in all four coordinates within the chosen frame of reference. 3. This result appears to me under all circumstances of massive importance for our idea of the physical nature of the gravitational field. For we either have to dispense with qualifying **t** – defined by (2) – as energy components of the gravitational field; thereby, however, the importance of the 'conservation principles' (see A. Einstein I.c) would collapse, and the task would arise to secure this integral part of the foundations in a new way. – If, instead, we hold on to the terms (2), then our calcucation teaches us that real gravity fields do exist (i.e., fields that cannot be 'transformed away'), with vanishing energy components, or, more precisely, energy components that can be 'transformed away'*; ..."

The latter of the two alternatives has to be dismissed: If there were true gravity fields, that is, gravity fields that cannot be "transformed away", these fields would, as true force-fields, have to have energy, that is, energy with components that cannot be transformed away, and would, because of the equivalence of mass and energy, have to act as a source of gravitational field lines. But the Schwarzschild solution tells us that it is only the ordinary mass and energy **M** that acts as a source of gravitational field lines.

3) The total absence of a true gravitational force and the indispensabiliy of the idea of flowing spaces in General Relativity

a) Hence, the term **t** in (3) does *not* stand for the energy components of the gravitational field. Given that the gravitational field has no energy, there is only one explanation for the fact that an object in free fall is nevertheless gathering speed: All gravitational fields can, without exception, be "transformed away" by the recognition that there are space volume elements in the vicinity of a gravitating mass that are permanently emerging and flowing towards the center of the gravitating mass. A second possibility, namely the existence of some unknown dark force that is acting on the falling object, can be dismissed for the following reason: If such a force existed, a freely falling electric point charge would, in the rest frame of that charge, have an electric field whose shape is not spherically symmetrical. But this would, as will be shown below, contradict the relativity principle.

More precisely: According to the correct interpretation of the solution of Einstein's field equation for spherical masses found by Schwarzschild (and by Droste only a short time thereafter), namely (**tau** stands for the proper time of an observer who is sitting in the gravity field, **t** denotes the time of an observer who is at rest far away from the gravitating mass, **G** is Newton's gravitational constant, **c** is the speed of light, **r** denotes the "distance" between an observer and the center of the spherical, gravitating mass, with this distance **r** being circumference of a circle, divided by 2 pi; **theta** and **phi** are angles in the system of polar coordinates that are used)
(11)

$$d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu = (1 - \frac{2GM}{c^2 r})\, dt^2 - \frac{1}{c^2(1 - \frac{2GM}{c^2 r})}\, dr^2 - \frac{r^2}{c^2}(d\theta^2 + \sin^2\theta\ d\varphi^2) \quad ,$$

there is no gravitational force. Instead, new space volume elements are steadily popping up in space out of nothing, so that space is permanently flowing towards the surface of the spherical mass, in order to disappear somewhere in its interior. The velocity of a volume element of space is increasing while the volume element is approaching the surface of the spherical mass. Objects floating in space take part in that accelerated flow.

Thereby all gravity fields are capable of being "transformed away" (without the introduction of flowing or streaming space, spherically symmetrical gravitational forces can undoubtedly be transformed away only locally, but not ubiquitously).

b) An illustration of the flow of space volume is provided by a modification of the Schwarzschild-metric, namely by replacing **$2MG/c^2r$** with **H^2R^2/c^2** , so that the metric can be applied to an expanding de-Sitter-universe characterized by a Hubble-constant **H** (=escape velocity, divided by distance) which is constant both in space and in time. The replacement is achieved by setting all components of the tensor **T** appearing in Einstein's field equation equal to zero, and by setting Einstein's additional term on its left side, that is the summand that contains the cosmological constant **lambda** as a coefficient, no longer equal to zero, but by giving **lambda** a positive numerical value.

Then we get (**tau** is the proper time of a second observer far away from Earth and the Milky Way, **t** is the time of a first observer who sits on Earth, **R** is the distance between the two observers, measured as circumference of a circle around Earth, divided by 2 **pi**):
(12)

$$d\tau^2 = (1 - \frac{H^2R^2}{c^2})\, dt^2 - \frac{1}{c^2(1 - \frac{H^2R^2}{c^2})}\, dR^2 - \frac{R^2}{c^2}(d\theta^2 + \sin^2\theta\ d\varphi^2)$$

All points in space whose distance from the first observer is not too close are escaping toward the cosmic event horizon on straight lines. No force is exerted on objects, for instance, on galaxies, that sit at those points in space. Instead, the accelerated motion is brought about by a steady expansion of space, that is, by a permanent popping up of space volume elements between the galaxies [see A. Einstein, Relativity – The Special and the General Theory, Bonanza Books 1961, Appendix IV, p. 134: *"Namely, the original field equations admit a solution in which the 'world radius' depends on time (expanding space).*"].

With regard to the gravitational field of a spherical mass, the situation is just alike. Different from the expanding universe, though, the emerging space volume elements do not drift toward the cosmic event horizon, but to the surface of the spherical mass (or, if the spherical mass constitutes a Black Hole, to the Schwarzschild-horizon) (see H. Reichenbach, The Philosophy of Space and Time, Dover Publ. 1958, § 36, Fig. 41, p. 226), in order to disappear into nothingness in the interior of the spherical mass. Note that according to the

Schwarzschild-equation, stationary, radially oriented meter sticks and stationary clocks in gravity fields behave exactly like meter sticks and clocks in Special Relativity do in case they are moving (along straight lines) at a velocity that is equal in amount to the escape velocity at the considered location in the gravity field. The straight and unaccelerated motion in Special Relativity is, in General Relativity, substituted by the permanent motion of objects – that sit at constant distance **r** from the center of mass – relative to the flow of space which is passing by (at a velocity that depends on radial distance **r**).

c) According to the famous interpretation of Einstein's field equation by J.A. Wheeler, masses in space tell space how to curve, and the curvature of space tells masses how to move. In order to realize this immediately, one should return to (2). This equation describes a geodesic, that is, the path taken by a body in space that is left to itself. In case space is curved, the partial derivatives of the components of the metric tensor **g** are not all zero (saying that not all spatial derivatives of the components of **g** are zero is nothing but a more precise way of saying that space is curved). As a consequence, the first summand in (2), namely, the ordinary divergence of the energy-momentum tensor **T**, is different from zero. This means that, according to (2), the coasting body absorbs or gives off kinetic energy and momentum. According to Equation 2, this is solely due to the fact that the derivations of the components of **g** are not all zero. It is for this reason that gravity is said to be nothing but curvature of space. No curvature of space (that is, all spatial derivatives of the components of **g** being zero), no acceleration. But in this mathematical view of gravity, not a single word is being said as to whether space in the gravitational field is moving or is at rest, instead. However, the arguments displayed above compel us to accept that the curvature of space can result in accelerated motions of masses only if volumes of space *move* in the reference frame of a distant observer.

4) Empirical consequences of space that flows or streams

a) Thereby the gravitational force is completely and ubiquitously"transformed away" in the rigid reference frame of a far-away observer. The complete absence of any gravitational force is made evident especially when looking at electric charge that is falling in a gravity field. No electromagnetic radiation is generated. For a co-falling observer, the electrostatic field of a point charge stays spherically symmetrical. Otherwise the famous observer in a freely falling elevator cabin could, contrary to the relativity principle according to which the co-falling observer in the elevator cabin may consider himself as being in the center of a local inertial system and hence at rest, tell by means of a simple observation, namely by determining the shape of the electrostatic field of a charge at rest, whether he is falling in a gravity field, or whether he is far away from heavy masses, instead. (For the relativity principle as applied to a space ship coasting in free space – as another example of an inertial system – see R.P. Feynman, Lectures on Physics, Vol. 1, 1965, Chapter 15-4, page 15-6: "*The biologists and medical men sometimes say it is not quite certain that the time it takes for a cancer to develop will be longer in a space ship, but from the viewpoint of a modern physicist it is nearly certain; otherwise one could use the rate of cancer development to determine the speed of the ship!*")

If, as is required by the relativity principle, the electrostatic field stays spherically symmetrical for a co-falling observer, electromagnetic radiation is neither generated in the rest frame of the falling observer, nor in the rest frame of a far-away observer who is stationary with respect to the gravitating mass.

The undistorted spherically symmetrical shape of the electric field of the falling point charge is subject to empirical testing. Since that shape of the electric field is required by the relativity principle, it is implicitly contained in Einstein's field equation (that is based on the relativity principle and on the invariance of the local speed of light). That is to say: If the electric field were distorted, the theory of General Relativity would be disproved.

b) In comparison, things are very different when an electric charge is "falling" in an external *electric* field (in the absence of gravity), i.e., if the charge is accelerated by the electric field between two huge capacitor plates. Then electromagnetic radiation is generated which is felt by a far-away observer, and for the co-falling observer the shape of the electrostatic field of the charge is distorted and no longer spherically symmetrical (for a picture of the electric field lines of a formerly stationary point charge that has been accelerated, see E.M. Purcell, Electricity and Magnetism, 1st edition 1965, Chapter 5.7, Fig. 5.16, p. 164).

This difference between the two scenarios provides justification for claiming that it is not only the charge, but also the surrounding space volume itself that, in case of a spherical symmetry of the electrostatic field of a charge falling in a gravity field, is in accelerated motion in the reference frame of a distant observer.

c) It was Einstein himself who, a few years prior to his death, mentioned the possibility of moving space volumes (A. Einstein, Relativity – The Special and the General Theory, Bonanza Books 1961, Appendix V – supplemented in 1952 by Einstein –, pp. 138, 139):

"*When a smaller box is situated, relatively at rest, inside the hollow space of a larger box S, then the hollow space of s is a part of the hollow space of S , and the same 'space', which contains both of them, belongs to each of the boxes. When s is in motion with respect to S, however, the concept is less simple. One is then inclined to think that s encloses always the same space, but a variable part of the space S. It then becomes necessary to apportion to each box its particular space, not thought of as bounded, and to assume that these two spaces are in motion with respect to each other.*

Before one has become aware of this complication, space appears as an unbounded medium or container in which material objects swim around. But it must now be remembered that that there is an infinite number of spaces, which are in motion with respect to each other. The concept of space as something existing objectively and independent of things belongs to pre-scientific thought, but not so the idea of the existence of an infinite number of spaces in motion relatively to each other. This latter idea is indeed logically unavoidable, but is far from having played a considerable role even in scientific thought."

The importance of this Appendix has recently been stressed by C. Rovelli (The Order of Time, 2017, p. 67, footnote).

5) The indispensability of “dark energy” in General Relativity

a) If it is taken for granted that the sum of kinetic and potential energy of a system of gravitating bodies is constant over time, and if it is also taken for granted that the gravitational field cannot have or give off any energy, it follows that an increase in the total energy of a system goes along with an appropriate decrease in some other form of energy (and not in energy of a gravitational field). One should keep in mind that the seat of potential energy of a test body is never in the test body itself; when the potential is realized, energy disappears elsewhere, e.g. in the interior of some field.

In our special case, however, one should note that the other form of energy which is involved here comes into play only when the falling test body hits the surface of the gravitating mass, so that thermal energy is generated. As long as the test body is in free fall, its kinetic energy – and hence its mass – is the same as it was when the test body was at rest outside of the gravity field, provided one is using floating coordinates in the vicinity of a gravitating mass. In other words: The mass of freely falling bodies does not increase, quite different from the mass of a body that is gathering speed in flat spacetime. The absence of any relativistic mass of a body falling in a gravitational field has been acknowleged by textbooks, but it has not been recognized that the reason for this absence lies in the fact that space itself is in accelerated motion near a gravitating mass.

If, in contrast, coordinates are used that do not participate in the flow of space near massive objects (but which are immovable in the frame of reference used), the lack of an increase in true kinetic energy of the falling body is hidden from view, as the freely falling body is gathering speed and is therefore increasing its apparent kinetic energy in that rigid frame of reference.

No matter which of the two types of coordinates are used, General Relativity thus cannot dispense of a “dark energy reserve” for the description of everyday phenomena in nature (including Newton’s apple). Given that the gravity field holds no energy, the second term in (2) cannot, as Einstein asserted, be an expression of the energy of the gravitatonial field (that is being converted into ordinary energy or vice versa), but must be an expression of the energy of a hidden and thus “dark” reservoir of energy.

The seat of that “dark energy” cannot be found in three-dimensional space. Otherwise that energy would contribute to the curvature of space and hence to the intensity of the gravitational “force”. Instead, one cannot do without the assumption of a fourth spatial dimension. It is in that fourth direction where the bulk of all “dark energy” must be located.

Thereby Schrödinger’s vision (cited above) of “new foundations” of the energy conservation principle – that are required as soon as one is realizing that the gravity field is void of energy – is coming true.

b) Of course, in order to solve the task of quantifying the amount of energy absorbed or given off by the reserve of “dark energy”, one is free to act as if the gravity field were in possession of a non-vanishing energy whose density is proportional to the negative square of the gravitational intensity in space. Nevertheless, one should always be aware of the fact that during a Helmholtz-contraction of a hollow gravitating sphere (by which, in theory, an infinite amount of mechanical work can be gained according to Newtonian physics), the field-free (!) space in the interior does not, contrary to common belief, possess a gravitational field with an infinite energy density. At that location, objects are rather in communication with an inexhaustable reserve of ‘dark energy’.

c) The cosmological constant **lambda**, which has been correctly regarded as an expression of “dark energy”, stands for nothing but that small fraction of “dark energy” which is *not* located in the direction of a fourth spatial dimension. In other words: The spatial position vector of this small part of “dark energy” points in the three familar spatial directions only, whereas the fourth component of this position vector is (practically) zero.

6) The nature of weight felt by statonary objects in gravity fields

a) For a distant observer who uses a rigid coordinate system (frame of reference), a test body which is falling at (**r**-dependent) escape velocity in a gravity field of a heavy mass is hence stationary relative to a flowing space volume element that surrounds the test body (while the test body is *not* stationary relative to the rigid system of coordinates the distant observer is using).

A test body freely falling at a velocity less than the **r**-dependent escape velocity does not change its velocity relative to the moving space volume element by which it is surrounded at any moment in time.

A test body sitting on the surface of the gravitating, spherical mass is not experiencing a downward “force” of gravity, but is experiencing upward intermolecular, repulsive forces from the surface on which it is sitting. These upward , intermolecular forces (that are counteracted by the inertia of the test body) do not manage, though, to set the test body in motion with respect to the surface of the spherical mass, but they do manage to set the test body in accelerated motion relative to a space volume element that is passing by.

The situation is analogous to an observer in a capsule hundreds of millions of lightyears away that is connected to planet Earth (and the Milky Way) by a long tether. Due to the expansion of space, the tether is under permanent mechanical tension, and the observer in the capsule feels a force that he might interpret as gravity (his time **tau** is described by Equation 12). The observer would be wrong, though: It is not *gravity* that is acting on the capsule and its contents, but the force exerted by the tether.

IV) Discussion

But what if the test body is radially *rising* in free motion in the gravity field at (**r**-dependent) escape velocity? In that case, gravity in the reference frame of the distant observer can only be “transformed away” by assuming the existence of space volume elements that do not flow *towards* the gravitating mass, but *move away* from this mass (thereby reducing their velocity, which is always equal to the **r**-dependent escape velocity).

How can space flow towards the gravitating mass and also move away from it? One should recall that Einstein mentioned (in 1952) the possibility of numerous, different velocities of space volumes as “logically unavoidable”, without, however, providing examples. If one assumes that space volume elements are permanently flowing toward the gravitating mass even when there is no test body in the gravity field, one is compelled to rate the free *rise* of space volume elements as being a result of a partial time reversal.

There is no other way of transforming the gravitational force away. The “transforming away” of the gravitational force, in turn, is indispensable for explaining how the mass that is freely rising in a gravitational field is being decelerated without transferring energy to the gravitational field (which is impossible due to its zero energy density) or to the energy reservoir of some dark, decelerating force (which would, in conflict with the relativity principle, lead – in the reference frame of the rising mass – to a deformation of the spherically symmetrical shape of the electric field the rising mass shall be imagined to be in possession of).

A partial time reversal, that is, the encounter of the “big arrow of time” with a much smaller one, is known in principle. The existence of anti-matter and also of hole conduction in semiconductors can be described in this fashion. However, so far only phenomena that occur on the *microscopic* scale have been candidates for representing “small arrows of time”. This limitation is no longer justified. As a consequence, the concept of a definite, uniform direction of time as something intrinsic to the macroscopic objects is rigorously destroyed.

Since gravitation is not a true force, and since the gravitational field has no energy (quite different from an electric or magnetic field), it is hardly conceivable that, on the level of quantum mechanics, gravitation is put into effect by particles, that is by gravitons.

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