Time reversals and loop-shaped world lines (yielded by the cosmic variant of the Schwarzschild solution) of objects or persons crossing the cosmic event horizon

by Andreas Trupp
andreas@andreastrupp.com

Abstract: The Schwarzschild solution of Einstein's field equation comes in two variants, one for a single spherical mass (Black Hole), and one for the universe as a whole. The cosmic variant represents a special form of expanding De-Sitter space (and is a good match with cosmic reality). As is shown, time-reversed world lines of objects and persons exist beyond the cosmic event horizon in that variant. This applies even to ourselves as humans: In the reference frame of a distant galaxy beyond our cosmic event horizon, the death of a human being on earth precedes her birth. Moreover, we all exist twice at the same time in that frame of reference. Last but not least, it is shown that traveling objects or persons are capable of crossing the cosmic event horizon back and forth. Thereby loop-shaped world lines of these traveling objects are inevitable. Similar possibilities exist in the context of Black Holes. The widespread belief of the contrary is based on a wrong method of using Kruskal charts. Only the correct use brings Kruskal-charts in harmony with the Schwarzschild equation, and is capable of explaining the observed fact that two Black Holes may interact with each other. Such observations reveal that the causal front reporting a change in gravitational "force" generated by an approaching Black Hole reaches the central mass of a second Black Hole and changes the path of that mass. In turn, the causal front carrying a report of the latter event makes it from the interior of the second Black Hole to the outside. All this is done within a finite amount of time, obviously similar in amount to that of Newtonian physics.

Key words: General relativity, Schwarzschild solution, Kruskal charts, cosmic event horizon, De-Sitter space, time travel, time reversal, closed world lines, black hole.

0. Introduction

Special Relativity has revealed that the temporal ordering (earlier/later) of two point-events is frame-dependent and hence relative. However, it has been believed that this relativity of a temporal ordering holds only true for two events which occur far away from each other in space so that there cannot be any causal interaction between them (given a causal front cannot spread at a speed faster than the local speed of light). The intervals between such two events have therefore be called "spacelike" intervals.

But two point events may occur so close to each other (and with a temporal distance from each other that is large enough) that there can be a causal interaction between them. The intervals between such two events are called time-like intervals. Birth and death of a person is an example of a pair of these events. It had been believed that there is no place in Relativity for the existence of reference frames in which time-like intervals between two events are relative in their temporal ordering (with the exception of exotic universes like the one mentioned by K. Gödel, see below). If the temporal orderings of such time-like events were relative, a person's death would precede her birth in a special frame of reference. Under some additional assumptions, a person could even meet her former self.

It was K. Gödel who was successful in showing that General Relativity allows for such closedness of world lines, but he had to resort to a strange hypothetical universe very different in geometry from ours. This article will show that General Relativity allows for time-reversals and loop-like world lines of persons and objects in our familiar universe as well. Not even such exotic things like ordinary Black Holes are required.

I. The world line of an escaping galaxy with a time-reversed section according to the Schwarzschild solution

a) The world line of an escaping galaxy

As is commonly known, the (outer) Schwarzschild solution of Einstein's field equation comes in two versions: The first one deals with a spherically shaped mass. The second one deals with the visible universe as a whole. It describes a De-Sitter-universe, in which dark energy prevails over matter (so that matter is negligible), and in which the Hubble-constant is invariant over space and time. A neighboring galaxy (that is, a galaxy neighboring our Milky Way) which is not gravitationally bound to any other galaxy will pick up speed as a result of the expansion of space, and will eventually reach the fixed cosmic event horizon several billion lightyears away. Just in front of the cosmic event horizon, the local escape velocity of that escaping galaxy, judged from the perspective of a local observer who is connected to the center of the distant Milky Way by an imagined, extremely long rope (and is thus standing still with respect to the Milky Way), is almost **c**.

The escaping galaxy will eventually cross the cosmic event horizon (that is attributed to the reference frame of the Milky Way). But apparently, it will do so only in the reference frame of that escaping galaxy, where this invisible borderline will rush by at a speed of **c** (just as other galaxies' cosmic event horizons are rushing past us here on Earth every second at velocity **c**). From the perspective of the Milky Way, this galaxy is practically not moving at all, but has got "frozen" in front of the cosmic event horizon, just as freely falling Alice gets "frozen" in front of the Schwarzschild horizon of a black hole from the perspective of distant Bob.

More precisely: The Schwarzschild solution for any spherically symmetric arrangement is (presuming spatial displacements occur in the radial direction only): $\mathbf{dtau^2 = f(r)dt^2 - dr^2/f(r)}$, with $\mathbf{f(r)}$ either being equal to $\mathbf{1\text{-}2a/r}$ or to $\mathbf{1\text{-}br^2}$. For the neighborhood of a spherical mass, the first alternative is used, and the constant **a** is set equal to $\mathbf{GM/c^2 = r_s/2}$, with $\mathbf{r_s}$ being the Schwarzschild radius, **G** being Newton's gravitational constant, **M** being the mass of the spherical body, **c** being the local speed of light. For a de-Sitter universe, the second alternative is used, with the constant **b** being set equal to $\mathbf{H^2/c^2}$, where **H** is Hubble's constant, that is, the increase in the galaxies' escape velocity with distance from the Milky Way. The numerical value of Einstein's cosmological constant determines the numerical value of Hubble's constant, and its sign determines the sign of Hubble's constant.

Hence, the cosmic variant of the Schwarzschild solution (de Sitter space) is:
(1)

$$d\tau^2 = (1 - \frac{H^2r^2}{c^2})\, dt^2 - \frac{1}{c^2(1 - \frac{H^2r^2}{c^2})}\, dr^2 - \frac{r^2}{c^2}(d\theta^2 + \sin^2\theta\ d\varphi^2)$$

The distance **r** denotes circumference of a circle around the center of the Milky Way divided by **2 pi**; time **t** is the time measured in the Milky Way; time **tau** is the time measured in the escaping galaxy.

In that variant, the role of a Schwarzschild horizon of a black hole is replaced by the cosmic event horizon. Different from the Schwarzschild horizon of a black hole, the cosmic event horizon is a relative thing, and every galaxy has one of its own. The Milky Way's cosmic event horizon is located at a distance of $\mathbf{r_{horiz}}$**= c/H** = 14,000,000,000 lightyears.

If Newton's physics were applicable to expanding cosmic space, the escape velocity of a galaxy as a function of distance from the Milky Way would be:
(2)

$$v_{newton} = Hr$$

According to (1), that Newtonian escape velocity (measured in the reference frame of the Milky Way) is reduced by two factors. These factors are the dilation of distant stationary clocks (stationary with respect to the Milky Way, that is, tied to the Milky Way by an imagined long rope) on the one hand, and the shortening of distant stationary meter sticks on the other hand. This leads to:
(3)

$$v_{escape} = \frac{dr}{dt} = Hr(1 - \frac{H^2r^2}{c^2}) \Rightarrow dt = \frac{1}{Hr(1 - \frac{H^2r^2}{c^2})}\, dr$$

And therefore (for an escape path of a galaxy that extends beyond the Milky Way's cosmic event horizon):
(4)

$$t(r) = C + \Delta t = C + \int_1^2 dt = C + \int_{r=r_0>0}^{r>r_{horiz}>r_0} \frac{1}{Hr - \frac{H^3r^3}{c^2}}\, dr = C + \left[-\frac{\ln(|\frac{c^2}{r^2} - H^2|)}{2H}\right]_{r=r_0>0}^{r>r_{horiz}>r_0}$$

The distance $\mathbf{r_0}$ is the distance of the considered galaxy at **t**=0 (given the constant **C** is set to zero). The (improper) integral is not divergent between the limits set (although the integrand is infinite at **r=**$\mathbf{r_{horiz}}$**= c/H**)!

The world line of the escaping galaxy as determined by (4) is shown in Fig. 1. In that diagram we have set **C=0**, **c=1**, **H=1**, so that **r** is expressed in dimensionless units or multiples of $\mathbf{r_{horiz}}$

(or in dimensionless units of 14 billion lightyears), **t** is expressed in units of time required to reach the cosmic event horizon by a light signal if the speed of light did not slow down (or in units of 14 billion years).

At $\mathbf{r}=\mathbf{r}_{\mathbf{horiz}}$ (= **c/H** =1), the time interval **Delta t** (in the Milky Way's frame of reference) that has elapsed since a nearby galaxy – which, at the beginning of the interval, was in the cosmic neighborhood of the Milky Way – has eventually reached the cosmic event horizon, is infinitely large according to (4). All motions in the interior of the escaping galaxy – and the escaping galaxy itself – get "frozen" in the reference frame of the Milky Way at spatial positions where **r** is only differentially smaller than $\mathbf{r}_{\mathbf{horiz}}$.

Nevertheless, the world line of that galaxy (in the **t,r**-diagram, that is, in the reference frame of the Milky Way) has a negative slope for any value of **r** larger than $\mathbf{r}_{\mathbf{horiz}}$. That means: Time is running in reverse at those distances. That is to say: Judged in the Milky Way's frame of reference, the clocks in the escaping galaxy (that is beyond the Milky Way's cosmic event horizon) are running backward. Moreover, the escaping galaxy exists twice at the same time in the reference frame of the Milky Way, provided the distance (of its first copy, i.e., the copy that finds itself on the near side of the horizon) from the Milky Way is roughly more than 3/4 of the invariant distance $\mathbf{r}_{\mathbf{horiz}}$ to the Milky Way's cosmic event horizon (see Fig. 1, generated online at www.integralrechner.de).

(At very far distances beyond the cosmic event horizon, the escape velocity of that galaxy, in the Milky Way's frame of reference, is approaching infinity. This is due to the fact that time shown by clocks which are stationary with respect to the Milky Way is contracted and meter sticks which are stationary with respect to the Milky Way are dilated at those distant locations beyond the cosmic event horizon.)

It is worth mentioning (as has been pointed out by L. Susskind) that, at every second, we are crossing some other galaxy's cosmic event horizon (which is rushing past us at velocity **c**). Consequently, our own world line is, at every moment that follows for us, time-reversed in the reference frame of that distant galaxy.

b) The world line of a galaxy in case cosmic expansion is succeeded by contraction

In order to convince ourselves of the fact that an escaping galaxy is capable of crossing the cosmic event horizon in the reference frame of the Milky Way (so that the continuation of the graph at regions of $\mathbf{r}>\mathbf{r}_{\mathbf{horiz}}$ is not just an artefact beyond the validity range of the Schwarzschild equation), we imagine the following thing to happen: We imagine that the expansion of the universe is succeeded by a contraction. One can simply assume that all motions of space and galaxies are reversed like the motion of a ping-pong ball hitting a wall (the galaxies wouldn't feel any acceleration, as they would stay embedded in cosmic space). In the cosmic version of the Schwarzschild solution, the sign of Hubble's constant would have to be altered from positive to negative. That's all.

In other words: We replace **+H** by **-H**. Then (4) describes the approach of a distant galaxy – and not its escape. Eventually, after billions of years, the galaxy that began its trip when it had been located close to the Milky Way is back at the location close to the Milky Way where it once had been. Given that all processes in nature are reversible, such an assumption is permissible. But this scenario leaves no room for any doubts concerning the reality of a crossing of the cosmic event horizon by an escaping galaxy even in the reference frame of the Milky Way. [In the spherical-mass (black hole) variant of the Schwarzschild solution, the analogue of this scenario is the traverse of a black hole by a freely falling and the freely rising test object.]

We hence have to acknowledge that there are time-like events, for instance the birth and death of ourselves on planet Earth, whose temporal order is reversed in another frame of reference. (In Special Relativity, only the temporal order of *space-like*, that is, causally unrelated events depends on the reference frame used.)

II. The world line of a radio signal sent off from an escaping galaxy beyond our cosmic event horizon

Moreover, we find that a distant, escaping galaxy beyond our cosmic event horizon can send radio signals that cross the cosmic event horizon and will eventually reach us, as will be shown in the following.

In a photon's rest frame, time does not exist. As a consequence, when applying (1) to a photon, the left-hand side of the equation is set to zero. We then get:
(5)

$$v^2_{photon} = \frac{dr^2}{dt^2} = c^2\left(1 - \frac{H^2r^2}{c^2}\right)^2 \Rightarrow dt = \pm\frac{1}{c\left(1 - \frac{H^2r^2}{c^2}\right)}dr$$

and hence (for an incoming photon, that is, a photon sent off from the escaping galaxy beyond our cosmic event horizon):
(6)

$$t(r) = C + \int_{r(t)>r_{horiz}>r_0}^{r(t)=r_0=0} dt = C + \int_{r>r_{horiz}>r_0}^{r=r_0=0} -\frac{1}{c\left(1 - \frac{H^2r^2}{c^2}\right)}dr = C + \frac{\ln(|Hr+c|) - \ln(|Hr-c|)}{2H}$$

The world line of the incoming photon is shown in Fig. 2 if the red graph in the third quadrant is imagined to be flipped over the vertical **t**-axis of the diagram into the fourth quadrant, or if the left half of the chart is used.

In case the limits of the integral are interchanged, (6) yields the world line of an *outgoing* photon. That world line can be seen in the first quadrant of Fig. 2 (generated online at www.integralrechner.de). Hence, the spatially fixed cosmic event horizon of the Milky Way is penetrable in both directions in the Milky Way's frame of reference.

III. Loop-shaped world lines of objects crossing the cosmic event horizon back and forth

a) In the scientific literature, loop-shaped world lines have been believed to be confined to strange universes of the kind conceived of by K. Gödel (1949) (p. 447):

"*Every world line of matter occurring in the solution is an open line of infinite length, which never approaches any of it's preceding points again; but there also exist closed time-like lines. In particular, if P, Q are any two points on a world line of matter, and P precedes Q on this line, there exists a time-like line connecting P and Q on which Q precedes P; i.e., it is theoretically possible in these worlds to travel into the past, or otherwise influence the past.*"

We are now facing the recognition that such loop-shaped world lines can exist in our ordinary universe as well. The graph in the first quadrant of Fig. 2 (generated online at www.integralrechner.de) does not only represent the world line of a photon, but also a world line of an object with a non-zero rest mass (say, Alice) that has been accelerated to almost the speed of light. Although the proper time needed to reach the Milky Way's cosmic event horizon cannot be zero (as it is for a photon), it could theoretically be as short as a few days. Having crossed the Milky Ways's cosmic event horizon, the object can be imagined to undergo a perfectly elastic reflection by an obstacle co-moving with a floating galaxy. The world line of the reflected object would then be represented by the graph in the third quadrant when this graph is flipped over the vertical **t**-axis of the diagram and is being lifted thereafter (due to **C** being positive then).

We realize: The farther away from the Milky Way (beyond the Milky Way's cosmic event horizon) the elastic reflection of the traveling object takes place, the less time will pass in the Milky Way after departure before the object will be back home!

In any case, the world line of the object is intersecting itself somewhere between the Milky Way and its cosmic event horizon.

b) In the face of the now emerging grandfather's paradox, we find that General Relativity leads to two alternative outcomes: According to the first of these two results, returning Alice is prevented from destroying her former self by some mysterious mechanism which science has not yet detected. But alternatively, we could say that the phenomenon of a choice between alternatives – that humans think they are capable of making – requires and is thus proof of the existence of multiple worlds. Hence, when returning Alice decides to destroy her former self, that destruction happens in a parallel world only, so that Alice is acting on a *copy* of herself.

A similar situation is described by D. Deutsch (1997)(pp. 304, 305):

"*So when I travel to the laboratory's past, I find that it is not the same past as I have just come from. Because of this interaction with me, the copy of me whom I find there does not behave quite as I remember behaving. ... If this were physical timetravel, the multiple snapshots at each instant would be parallel universes. Given the quantum concept of time, we* *should not be surprised at this*."

IV. Analogous situations in and around Black Holes

1) The equation for a photon's trip both into and out of a Black Hole

Analogous situations exist when it comes to the spherical-mass variant of the Schwarzschild solution. The trip of a nearby galaxy – that escapes across the Milky Way's cosmic event horizon in order to reverse its path after the universe has traded its expansion for contraction – is exchanged for a trip across a black hole by an observer called Alice. The trip can be done in the form of a free fall from far away with an initial speed of almost zero, succeeded by a free radial rise, along an imagined shaft in the spherical mass. Such a trip (and the role of Kruskal charts that appear to make such trips impossible) is discussed by A. Trupp (2020).

A similar trip can be done by a photon (or an object with a non-zero rest mass previously accelerated to almost the speed of light). The photon can be imagined to fall into or rise from a Black Hole on a radial path. In a photon's rest frame, proper time does not exist. As a consequence, when applying the Schwarzschild equation to a photon, the left-hand side of the Schwarzschild equation is set to zero. We then get for a photon that is traveling towards or away from a Black Hole on a strictly radial path [see L. Susskind, A. Cabannes (2023), Eq. 40, p. 194):
(7)

$$v^2_{photon} = \frac{dr^2}{dt^2} = c^2\left(1 - \frac{r_s}{r}\right)^2 \Rightarrow dt = \pm\frac{1}{c\left(1 - \frac{r_s}{r}\right)}\, dr$$

Note that **r** is defined as "circumference of a circle (around the center of the spherical mass) divided by 2 pi", and is not what a stationary, radially-oriented tape measure would yield.

For a photon or an equivalent object that rises from the center of the spherical point-mass (at **t**=0, **r**=0) on a radial path, we thus get from (7):
(8)

$$t(r) = C + \int_{r(t)=r_0\to 0}^{r(t)>r_s} dt = C + \int_{r=r_0\to 0}^{r>r_s} \frac{1}{c\left(1 - \frac{r_s}{r}\right)}\, dr = C + \left[\frac{r_s \ln(|r-r_s|) + r}{c}\right]_{r=0}^{r>r_s}$$

[See the **t**,**r**-diagram – with **r** as the abscissa and **t** as the ordinate – in of Fig. 3 (generated online at www.integralrechner.de); the red graph in the right half of the diagram depicts the world line of the outgoing photon, that is, the graph which represents the function **t(r)**; the Schwarzschild horizon is at **r**=+1.] Again, the (improper) integral, that is, (8), is not divergent between the limits set (although the integrand is infinite at $\mathbf{r=r_s}$)!

If $\mathbf{r_s}$ (and not meter) is chosen as the (dimensionless) unit of spatial length, (8) turns into (if

C=0):
(9)

$$t = \int_{r(t)=r_0=0}^{r(t)\gg r_s=1} dt = \int_{r=r_0\to 0}^{r\gg r_s=1} \frac{1}{c(1-\frac{1}{r})} dr = \frac{\ln(|r-1|)+r}{c} \approx \frac{r}{c} \;(for\; r\gg 1)$$

If a photon or an equivalent object is *traversing* the Black Hole (and not just rising from the center), the result for **t** has to be doubled. The equation for the infalling photon would simply be (for **C**=0):
(10)

$$t = \int_{r(t)\gg r_s=1}^{r(t)=r_0=0} dt = -\int_{r(t)=r_0=0}^{r(t)\gg r_s=1} dt = -\int_{r=r_0\to 0}^{r\gg r_s} \frac{1}{c(1-\frac{1}{r})} dr = \frac{-\ln(|r-1|)-r}{c} \approx -\frac{r}{c} \;(for\; r\gg 1)$$

Given **C**=0, the photon reaches **r**=0 at **t**=0. Its departure was at **t=-r/c**, that is, prior to **t=0**, provided the spatial location of the departure was at $\mathbf{r>>r_s}$.

2) Why Kruskal charts do not object to the possibility of a photon's trip out of a Black Hole

a) But how can it be that all textbooks assert that it is impossible even for a photon to cross the Schwarzschild horizon from the inside to the outside?

Textbooks convert the Schwarzschild equation which uses polar coordinates into a so-called Kruskal chart that uses special coordinates. When using a Kruskal chart, **t** and **r**, which appear in the equation of the Schwarzschild solution for a spherical mass, are substituted by new variables **T** (ordinate) and **X** (abscissa) according to the following transformation rule (**c** is set to unity and does not appear):
(11)

$$T = \pm\sqrt{(\frac{r}{2GM}-1)}\; e^{r/4GM} \sinh(\frac{t}{4GM})$$

$$X = \pm\sqrt{(\frac{r}{2GM}-1)}\; e^{r/4GM} \cosh(\frac{t}{4GM})$$

The above transformation is used for regions outside the Schwarzschild radius of Black Holes, that is for $\mathbf{r>r_s}$.

For regions inside the Schwarzschild radius ($\mathbf{r<r_s}$), the following transformation is used:

(12)

$$T = \pm\sqrt{\left(\frac{r}{2GM} - 1\right)} \;\; e^{r/4GM} \cosh\left(\frac{t}{4GM}\right)$$

$$X = \pm\sqrt{\left(\frac{r}{2GM} - 1\right)} \;\; e^{r/4GM} \sinh\left(\frac{t}{4GM}\right)$$

When confining our attention to the area between the positive **X**-axis and the positive **T**-axis of the new chart (with **T** as the ordinate and **X** as the abscissa), we find: "Lines of same **r**" arise from the horizontal **X**-axis at right angle, and form curves that assymptotically approach the diagonal (between the vertical positive **T**-axis and the horizontal positive **X**-axis). The diagonal line is identical with the "line of same **r=**$\mathbf{r_s}$". There are other curved "lines of same **r**" which originate on the vertical **T**-axis at right angle, and form curves that asymptotically approach the diagonal from the other side. The uppermost of these lines is the "line of same **r**=0". Moreover, lines of "same coordinate time **t**" (=Bob's time) are straight lines that all originate at the origin of coordinates. The horizontal positive **X**-axis coincides with the line of "same coordinate time **t=0**". The diagonal line (which represents the line of "same **r=**$\mathbf{r_s}$") coincides with the line of "same coordinate time **t= +inf** ". The vertical positive **T**-axis coincides with the line of "same coordinate time **t=0**" (as does the horizontal positive **X**-axis).

The world line of a radial light pulse forms an angle of 45° with the **X**- and the **T**-axis in all quadrants of the diagram. Hence, the world line of an infalling photon that is generated at **t**=0 orginates on the positive horizontal **X**-axis of the chart, given that **r**, that is, the spatial location where the photon is "born"**,** is larger than $\mathbf{r_s}$. The photon's world line has a slope of 45° and intersects the diagonal at right angle. The coordinate time **t** (Bob's time) of this crossing of the Schwarzschild horizon is positive infinity. The photon's straight world line then intersects "lines of same coordinate time **t**" with declining numerical values until it reaches the curved line that stands for **r**=0. All this is a perfect match with what (10) yields for an infalling photon. In order to visualize the infalling photon's world line – as it presents itself both in a **T,X**-Kruskal chart and also according to (10) –, the red graph shown in Fig. 3 (**t**,**r**-diagram) must be flipped over the horizonal **r**-axis (only the right half of the diagram is of interest here).

Now comes the crucial point: It is commonly believed that the world line of an outgoing photon that is "born" in the interior of a Black hole at **r<**$\mathbf{r_s}$ can be pictured as a straight line in the same sector of the Kruskal chart which has been used for representing the *infalling* photon. That sector was the one defined by the positive horizontal abscissa (positive **X**-axis) and the positive vertical ordinate (positive **T**-axis). The result is shown in Fig. 4 (see below). The world line of the "outgoing" photon ends up at **r=0** and never approaches the Schwarzschild horizon at **r=**$\mathbf{r_s}$.

L. Susskind / A. Cabannes (2023) (Fig. 17, p. 232/233) recently expressed this widespread belief as follows:

"*Remember, in the coordinates that we are using, light moves with a 45° angle. Therefore light cannot escape from the upper quadrant in figure 17. All it can do is eventually hit the singularity* [at r=0]. *And anything that is moving slower than the speed of light has a slope closer to the vertical, and will also hit the singularity. Consequently, anybody who at some point is in the upper quadrant is doomed. ... Figure 17, and its variants figures 15 and 16, are pictures you should familiarize yourself with, until they no longer have any secrets. If you want to understand and be able to resolve weird paradoxes about who sees what in the black hole, I recommend that you always go back to these diagrams.*"

But this is clearly at odds with what our equations (8) or (9) (valid for an outgoing photon) are telling us. They tell us that the photon *does* approach the Schwarzschild horizon asymptotically. The world line given by (8) or (9) is shown in Fig. 3 (generated online at www.integralrechner.de) as the red graph (only the right half of the diagram is of interest; the photon is "born" at coordinate time **t**=0 at the spatial position **r**=0; the Schwarzschild horizon is at **r**=+1).

Moreover, Susskind's statement:

"*And anything that is moving slower than the speed of light has a slope closer to the vertical,... "*

is incorrect. In order to realize this, we choose a point on the chart that sits somewhere between the diagonal and the vertical **T**-axis (in the sector of the Kruskal used for the respresentation of an inbound light pulse). More precisely: Our point shall have the coordinates **r**= 0.8 $\mathbf{r_s}$, **t**= 1.0 (with the "line of same **t**=1.0" being defined as halving the 45°-angle formed by the vertical **T**-axis and the diagonal). Supposing the world line of an inbound light pulse that originated at a point far away at **r>>**$\mathbf{r_s}$ runs through that point, the straight line that represents the infalling photon's world line forms an angle of 45° to the left of the vertical. When, in our imagination, we rotate that straight line clockwise (towards the vertical and beyond) with our fixed point on the chart as a foothold, the line does not stand for an object that is moving slower than a photon (into the past of coordinate time **t** and towards **r**=0), but faster! This is because of the following: The temporal coordinate **t** of points on the neighboring "line of same **r**=0.6" reached by our rotating straight line (that orginates at our fixed point **r**=0.8, **t**=1) increases in (positive) amount with increasing size of the angle which the rotating straight line is forming with 45°-line (representing the world line of an infalling photon) from which the rotation started. When the rotating straight line coincides with "line of same **t**=1.0" (that originates at the center of coordinates as all "lines of same **t**" do), it stands for the world line of an object with an infinite speed of (anti-) radial motion. When rotating the world line even further, the line stands for the world line an object that is moving at finite speed towards **r**=0; but now the object is reaching **r**=0 in the future (of coordinate time **t**), and no longer in the past.

Sofar, so good. Yet one has to realize that the world line of an object that does not move at all coincides with a "line of same **r**". In the sector of a Kruskal chart defined by the diagonal and the positive vertical **T**-axis, any short straight piece of a curved "line of same **r**" is farther away (in terms of angle size) from the vertical (in a clockwise direction) than just 45°. A straight line that is even a tiny bit farther away from the vertical than the "line of same **r**" thus represents a world line of an outbound object which is moving as slowly as one wishes it to do, that is, below the local speed of light **c** , and is approaching the Schwarzschild horizon. Eventually, the outbound object can cross the Schwarzschild horizon. This constitutes a contradiction with the previously obtained result (according to which any world line is doomed to end at **r**=0).

The error is rooted in the disregard of the following rule: The sector defined by the positive horizontal abscissa and the positive vertical ordinate of the Kruskal chart can only be used for *inbound* objects (traveling into the Black Hole), not for *outbound* ones. For *outbound* light signals or other objects, a different sector of the chart has to be used: the sector defined by the negative vertical ordinate (negative **T**-axis) and the positive horizontal abscissa (positive **X**-axis). Then the result given by (8) or (9) for an outgoing light pulse (see Fig. 3) is reproduced by the Kruskal chart (as it should). Moreover, the world line of an outgoing photon then is symmetrical with respect to the world line of an *in*going photon, just as it should be due to the principle of reversibiliy of any light path. The common but wrong interpretation of Kruskal charts is incompatible with the principle of "reversibility of any light path". By contrast, the correct use of Kruskal charts gives due consideration that principle.

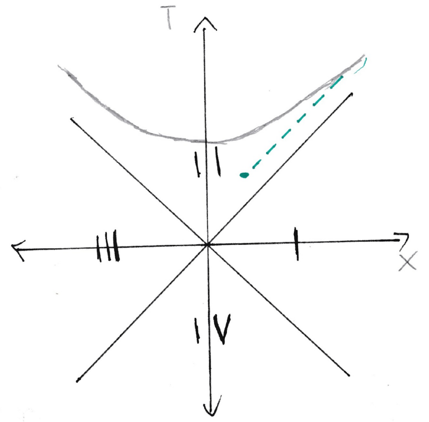

FIG 4: When (erroneously) using that sector of a Kruskal chart which is defined by the positive horizontal X-axis and the positive vertical T-axis for a representation of the world line of an outbound light pulse originating beyond the Schwarzschild horizon, that light pulse cannot but eventually hit the singularity at r=0. However, in order to do things right, not this sector, but the sector defined by the positive horizontal X-axis and the negative vertical T-axis has to be used for a representation of the world line of an outbound light pulse.

b) Moreover, a simple thought experiment reveals that an impossibility for any outbound light signal to cross the Schwarzschild horizon would violate some laws of nature:

Imagine Alice finds herself in the interior of a pencil-shaped spacecraft. The spacecraft shall be in free fall towards the Schwarzschild horizon head first. When the spacecraft – which, according to the relativity principle, may consider itself as being at rest – reaches the Schwarzschild radius, the invisible, moving wall which constitutes the Schwarzschild horizon is whizzing through the spaceship from head to tail at a speed – in Alice's (= in the spacecraft's) frame of reference – much lower than that of light. The low speed of motion of the invisible wall (in Alice's frame of reference) is due to the presumed fact that the spaceship shall have started its free fall not from afar, but from somewhere in the gravity field of the spherical body.

When a freely falling observer is passing by an observer who is at rest in the gravity field, both observers measure the same (absolute) velocity of motion of the other observer. This, too, is a consequence of the relativity principle. If the observer who is at rest in the gravity field sits close to the Schwarzschild horizon, and if the free fall of the other observer started far away, the falling observer's velocity of free fall, measured by the observer at rest in the gravity field, is identical in absolute amount with the local escape velocity, and is therefore close to **c**. This is also the speed at which the falling observer (who considers himself or herself as being at rest) watches the other observer (who is at rest relative to the spherical mass and the gravity field) rush past him or her in his or her frame of reference.

But given the free fall did *not* start from afar, the falling observer's speed when passing by the observer who is at rest in the gravity field very close to the Scharzschild horizon (measured in the reference frame of that observer at rest in the gravity field) is much lower than the local escape velocity (which is close to **c**), and thus falls short of **c** by a large amount. So does the speed at which the Schwarzschild horizon is rushing past the falling observer (that is, the observer Alice in the freely falling spaceship) in his or her frame of reference.

Let us scrutinze the moment in Alice's time when the moving "wall" has reached the spaceship (which considers itself as being at rest) and has just gone by the ship's bow. In the interior of the pencil-shaped spaceship, a light signal shall be sent from the ship's bow to its stern. Given the speed of the progressing, invisible wall is much lower than **c**, the light signal will catch up with the wall and overtake it. It will thus reach the ship's stern prior to the arrival of the invisible wall.

We realize: In the interior of the spaceship, the moving Schwarzschild horizon was crossed from the inside to the outside by the light signal. In case that would not have happened, that is, in case the light signal had been slower than **c** in Alice's frame of reference, either the law of the invariance of the local speed of light, or the relativity principle (according to which Alice and the spaceship may consider themselves as being at rest) would be violated. But the exceptionless validity of these two principle is the basis of Relativity.

c) Consequently, contrary to what L. Susskind and A. Cabannes are saying (in accordance with all textbooks on Black Holes), their diagram 17 is not crucial for a correct understanding of what Black Holes are. Instead, it is responsible for a long-standing, complete misconception regarding Black Holes.

d) This misconception has led to another wrong conclusion. Any volume element of the central mass is believed to be inevitably crushed into a central singularity at r=0. This is because of the following reflection. The world line of a volume element of the central mass of the black hole must find itself somewhere between the world line of a ingoing photon and the world line of a photon that is unsuccessfully trying to leave the black hole. But each of these two world lines of a photon end up in the singularity at **r=0**. Then, however, it seems that the world line of a volume element of the central mass cannot but end up at the singularity, too. This is because that world line cannot but find itself somewhere between the two world lines of a photon, given all world lines originate at the same point on the chart.

But as soon as one realizes that the world line of an outgoing photon does *not* end up in the singularity at **r=0**, but extends into the exterior if the correct quadrant of the chart is used, one realizes that the world line of a volume element of the central mass does *not* end up at the singularity at **r=0**, but keeps its **r**-coordinate over time. In other words: The world line of the volume element coincides with a line of constant **r**.

3) What the inner Schwarzschild solution can tell about how to use Kruskal charts

We will now turn our attention to what is happening in the *interior* of a spherical mass. In order to do so, we have to use the inner Schwarzschild solution of Einstein's field equation. The inner Schwarzschild solution of Einstein's field equation reads:
(12a)

$$c^2 d\tau^2 = \frac{1}{4}\left(3\sqrt{1-\frac{r_s}{R_0}} - \sqrt{1-\frac{r^2 r_s}{R_0^3}}\right)^2 c^2 dt^2 - \frac{1}{1-\frac{r^2 r_s}{R_0^3}} dr^2 - r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

It applies to the interior of a sphere, which, in Bob's frame of reference, is homogeneously filled with matter. $\mathbf{R_0}$ is the radius of that spherical body (in Bob's frame of reference). Whenever $\mathbf{r_s < R_0}$, the radius $\mathbf{r_s}$ is no longer the radial distance at which the local escape velocity is **c**, but is simply equal to $\mathbf{2GM/c^2}$.

In case $\mathbf{r_s = R_0}$, the inner Schwarzschild solution turns into (presuming spatial motions only occur in a radial or anti-radial direction):
(12b)

$$c^2 d\tau^2 = \frac{1}{4}\left(-\sqrt{1-\frac{r^2}{R_0^2}}\right)^2 c^2 dt^2 - \frac{1}{1-\frac{r^2}{R_0^2}} dr^2$$

For two point events that occur at the same location in the interior of that sphere (**dr**=0), we thus get:
(12c)

$$\frac{d\tau}{dt} = -\frac{1}{2}\sqrt{1-\frac{r^2}{R_0^2}}$$

There is no singularity at **r=0**! At the center of the sphere (where no gravitational acceleration is present), local, stationary clocks are running half as fast as they do far away from the spherical mass, and they run in the opposite direction (inversed time)!

With no singularity at **r=0**, no object that tries to make it to the exterior on a radial path can be doomed to end up there. The Kruskal-chart simply isn't applicable here, as it is taylored for the outer Schwarzschil solution only, not for the inner one. But given that photons and other objects are capable of crossing the Schwarschild horizon from the interior to the exterior in case of $\mathbf{R_0=r_s}$, there is no reason to assume that they cannot do so also in case $\mathbf{R_0}$ is very small compared to $\mathbf{r_s}$, as is presupposed in the outer Schwarzschild solution.

It should be noted that R.P. Kerr, too, recently doubted that a central singularity could exist in Black Holes. He stated [Kerr (2023)]:

"*There is no proof that black holes contain singularities when they are generated by real physical bodies*."

The inner Schwarzschild solution provides the easiest way to prove that a central singularity does not exist (as was shown above).

4) The matching of proper time and coordinate time needed for freely falling objects to traverse a Black Hole from afar as another proof of the common misinterpretation of Kruskal charts

Let us now imagine that Alice is no longer traveling almost at the speed of light, but is moving towards the Black Hole in free fall from afar (having started with a speed of almost zero), and is freely rising from the Black Hole after having traversed the spherical mass through an imagined shaft. As is described by Trupp (2020), the coordinate time (that is, Bob's time who is at rest very far away from the spherical mass) needed for Alice's free-fall-free-rise traverse according to Newtonian physics (which, as can be shown, is also Alice's proper time **tau** in General Relativity) is given by the expression *below* the bar, whereas the coordinate time (Bob's time) needed for a traverse according to General Relativity is given by the expression *above* the bar (a factor of 2 has been cancelled above and below the bar):
(13)

$$\lim_{r\to\infty}\frac{\Delta t}{\Delta\tau} = \lim_{r\to\infty}\frac{\frac{r_s}{c}\left[\frac{\left(\frac{6r_s}{r}+2\right)\left|\left(\frac{r}{r_s}\right)^{3/2}\right|}{3} - \ln\left(\left|\sqrt{\frac{r}{r_s}}+1\right|\right) + \ln\left(\left|\sqrt{\frac{r}{r_s}}-1\right|\right)\right]}{\frac{2r_s}{3c}\left|\left(\frac{r}{r_s}\right)^{3/2}\right|}$$

$$= \lim_{r\to\infty}\frac{\frac{r_s}{c}\left[2\left|\left(\frac{r}{r_s}\right)^{1/2}\right| + \frac{2}{3}\left|\left(\frac{r}{r_s}\right)^{3/2}\right| - \ln\frac{\left|\sqrt{\frac{r}{r_s}}+1\right|}{\left|\sqrt{\frac{r}{r_s}}-1\right|}\right]}{\frac{2r_s}{3c}\left|\left(\frac{r}{r_s}\right)^{3/2}\right|} = 1$$

Ch.W. Misner, K.S. Thorne, J.A. Wheeler (1973)(§ 25.5, Equation 25.38, p. 667) would, for far-away starting points of a radial fall and rise, have come to the same result, if they had realized that their quotient

(14)

$$\underset{r\to\infty}{\underline{limes}}\ \frac{\frac{t}{2M}}{\frac{\tau}{2M}} = \underset{r\to\infty}{\underline{limes}}\ \frac{-\frac{2}{3}\left(\frac{r}{2M}\right)^{3/2} - 2\left(\frac{r}{2M}\right)^{1/2} + \ln\frac{\left(\frac{r}{2M}\right)^{1/2}+1}{\left(\frac{r}{2M}\right)^{1/2}-1}}{-\frac{2}{3}\left(\frac{r}{2M}\right)^{3/2}} = 1$$

of their expressions for **t** and **tau** – appearing in first and second line of their Equation 25.38 – approaches unity for very large **r**. The term above the bar in (13) can also be found in R.J.A. Lambourne, Relativity, Gravitation and Cosmology, Cambridge University Press, Chapter 6.2.2, Eq. 6.19, p. 181.

The fact that the quotient of **Delta tau** and **Delta t** approaches unity for large **r** is another proof that the crossing of the Schwarzschild horizon of a black hole from the interior to the exterior is possible. It hence also a proof of the massive flaw in the usual interpretation of Kruskal charts when it comes to Black Holes.

At the same time, it is proof of the fact that Einstein's interpetation of relativity principle (see below) can be deduced from the Schwarzschild solution of Einstein's field equation.

5) The matching of proper time and coordinate time needed for freely falling objects to traverse a Black Hole from afar as as a consequence of the relativity principle that forms a starting point of General Relativity

a) The matching of Alice's proper time and Bob's coordinate time is not a coincidence. One has to realize that Alice is allowed to consider herself at rest even when falling through a Black Hole. And this requires that Alice is not privileged with respect to any other observers who, too, are allowed to consider themselves as being at rest. This is the essence of the relativity principle. A. Einstein (2018) (p. 136) described this important aspect of the relativity principle as follows:

"*At that moment I got the happiest thought of my life in the following form: In an example worth considering, the gravitational field has a relative existence only in a manner similar to the electric field generated by magneto-electric induction. Because for an observer in free-fall from the roof of a house there is during the fall—at least in his immediate vicinity—no gravitational field. Namely, if the observer lets go of any bodies, they remain* *relative to him, in a state of rest or uniform motion, independent of their special chemical or physical nature. The observer, therefore, is justified in interpreting his state as being 'at rest.' The extremely strange and confirmed experience that all bodies in the same gravitational field fall with the same acceleration immediately attains, through this idea, a deep physical meaning. Because if there were just one single thing to fall in a gravitational field in a manner different from all others, the observer could recognize from it that he is in a gravitational field and that he is falling. But if such a thing does not exist—as experience has shown with high precision—then there is no objective reason for the observer to consider himself as falling in a gravitational field. To the contrary, he has every right to consider himself in a state of rest and his vicinity as free of fields as far as gravitation is concerned. The experimental fact that the acceleration in free-fall is independent of the material, therefore, is a powerful argument in favor of expanding the postulate of relativity to coordinate systems moving nonuniformly relative to each other.*"

If Einstein is correct, his postulate must be deducable from the Scharzschild solution of Einstein's field equation and hence from Einstein's field equation itself. This is what explains the sameness of **Delta tau** and **Delta t** for large **r**.

b) However, Alice's trip *does* take a tiny bit longer for Alice than for Bob. This phenomenon is due to the fact that, strictly speaking, Bob is *not* sitting in a perfect inertial system, given gravity does not completely vanish even far away from the spherical mass. In other words: Although the *quotient* of **Delta tau** and **Delta t** approaches unity for large **r**, the *difference* between **Delta tau** and **Delta t** does not.

This non-vanshing of the difference is accounted for by the following reflection: Bob, whose time is expressed by **t**, is not really outside of the gravitational field generated by the spherical mass (as has already been said). No object can complete defy that gravity, since the gravitational acceleration does not reduce to exactly zero anywhere. An increase in **r** (denoting Bob's position in space) by the factor 100 leads to an increase in **Delta t** by a factor of 1000. It also leads to a decrease in the weak gravitational time dilation that Bob is subject to. But that factor of decrease, which is equal to $[(1-\mathbf{r_s}/\mathbf{r_2})^{1/2} / (1-\mathbf{r_s}/\mathbf{r_1})^{1/2}]$, falls short of 1000. This is why the difference between **Delta tau** and **Delta t** must increase with **r**.

6) Observed interactions between two black holes as evidence of a wrong usage of Kruskal charts

In the recent past, interaction of black holes have been observed. They are assumed to be the cause of gravitational waves that have been measured. These gravitational waves were generated when two black holes merged by spiraling into each other.

Such observations reveal that the causal front reporting a change in gravitational "force" generated by an approaching Black Hole reaches the central mass of a second Black Hole and changes the path of that mass. In turn, the causal front carrying a report of the latter event makes it from the interior of the second Black Hole to the outside. All this is done within a finite amount of time, obviously similar in amount to that of Newtonian physics.

The causal front (that brings change to the local gravitational “force” at given, fixed locations) moves at the speed of light. Its world line therefore resembles that of a photon which falls into a black hole or leaves it the way described above. Such observations of merging black holes are thus evidence of the fact that the common interpretation of Kruskal-charts is wrong. And they are proof of the fact that the world lines of objects leaving a black hole are a physical reality. Then, however, time reversal within the Scharzschild radius is a physical reality, too.

V. Results

The results are the following:

– In the cosmic variant of the Schwarzschild solution (expanding space), time-reversed world lines of objects and persons exist beyond the cosmic event horizon.

– All persons and objects on earth exist twice in the reference frame of those galaxies which find themselves beyond the Milky Way’s cosmic event horizon.

– Traveling objects or even persons are capable of crossing the cosmic event horizon back and forth. Thereby loop-shaped world lines of these traveling objects are inevitable.

– Similar possibilities exist in the context of Black Holes. The first quadrant of a **T**,**X**-Kruskal chart may only be used for world lines of photons or other objects going *into* a Black Hole. As regards *outbound* photons or other objects, the fourth quadrant – and not the first one – of a Kruskal chart has to be used. This guarantees a complete reversibility of any light path into or out of a Black Hole.

References:

Deutsch, D. (1997), The Fabric of Reality, 1997;

Einstein, A. (2018), Doc 31: Ideas and Methods, II, The Theory of General Relativity, in: Janssen, Michel / Schulmann, Robert / Illy, József / Lehner, Christoph / Buchwald, Diana (eds.), The Collected Papers of Albert Einstein. Volume 7: The Berlin Years: Writings, 1918 – 1921 (English translation supplement) (Digital ed.), California Institute of Technology, retrieved 15 April 2018;

Gödel, K. (1949), An Example of a New Type of Cosmological Solutions of Einstein’s Field Equations of Gravitation, Reviews of Modern Physics, Vol. 21, 3 (1949), pp. 447-450;

Kerr, R. P., Do Black Holes have Singularities?, arXiv:2312.00841v1 [gr-qc] 1 Dec 2023;

Misner, Ch. W, Thorne, K.S., Wheeler, J.A. (1973), Gravitation;

Susskind, L., Cabannes, A. (2023), General Relativity – The Theoretical Minimum;

Trupp, A. (2020), Traversing a black-and-white hole in free fall and rise, Physics Essays, Vol. 33, 4 (2020), pp. 460-465.

Last revised: 09-Jan-2024