A ubiquitous, nearby reservoir of electromagnetic energy hidden in the fourth spatial dimension as a consequence of Kelvin's rule (for constant electric currents) and of the Poynting vector

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Abstract: When applying what is called Kelvin's principle to the elementary currents of two permanent magnets that attract each other, an apparent energy paradox appears. For Kelvin's principle says that when constant electric currents are displaced with respect to one another, the mechanical work yielded as a result of the action of magnetic forces is equal in amount to the increase (not decrease) in the energy of the total magnetic field. The energy provided by the power supply in order to keep the currents constant is thus twice as large as the mechanical work yielded during the displacement of the current-carrying wires. But when dealing with permanent magnets and their polarization currents, there is still the yield of mechanical work and also the increase in energy of the total magnetic field, but no such thing as a visible power supply. In this article, things are analyzed by using the Poynting vector as an instrument. As a result, the topological assumption of a hidden reservoir of energy sitting in the direction of a fourth spatial dimension turns out to be indispensable in order to save the principle of local conservation of energy and of action by contact. A recognition of this kind was foreshadowed by Mie 100 years ago, who postulated that, in certain, but nevertheless common situations, energy flowed into ambient space out of the particles themselves both in the gravitational and the electromagnetic case. © *2023 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-36.3.287]

Résumé: En appliquant ce qu'on appelle le principe de Kelvin aux courants élémentaires de deux aimants permanents qui s'attirent, un apparent paradoxe énergétique apparaît. Car le principe de Kelvin dit que lorsque des courants électriques constants sont déplacés les uns par rapport aux autres, le travail mécanique produit à la suite de l'action des forces magnétiques est égal en quantité à l'augmentation (et non à la diminution) de l'énergie du champ magnétique total. L'énergie fournie par l'alimentation pour maintenir les courants constants est donc deux fois plus grande que le travail mécanique fourni lors du déplacement des fils conducteurs de courant. Mais lorsqu'il s'agit d'aimants permanents et de leurs courants de polarisation, il y a toujours le rendement du travail mécanique et aussi l'augmentation de l'énergie du champ magnétique total, mais pas une alimentation visible. Dans cet article, les choses sont analysées en utilisant le vecteur de Poynting comme instrument. De ce fait, l'hypothèse topologique d'un réservoir d'énergie caché situé en direction d'une quatrième dimension spatiale s'avère indispensable pour sauver le principe de conservation locale de l'énergie et d'action par contact. Une reconnaissance de ce genre a été annoncée par Mie il y a cent ans, qui postulait que, dans certaines situations, l'énergie s'écoulait dans l'espace ambiant à partir des particules elles-mêmes, tant dans le cas gravitationnel qu'électromagnétique.

Key words: Electromagnetic Energy; Kelvin's Rule; Poynting Vector; Fourth Dimension; Action and Reaction; Force and Counterforce; Brane Universe; Mach's Principle.

I. INTRODUCTION—KELVIN'S PRINCIPLE AND THE ENERGY PROBLEM ARISING FROM THIS PRINCIPLE WHEN TWO PERMANENT MAGNETS ARE APPROACHING ONE ANOTHER

(a) Imagine that two equal, circular wire loops carry a stationary current (maintained by a power source) of constant current strength. Let the two wire loops approach each other until they eventually sit side by side. During the linear displacement of the two loops, they had been attracting each other (given their right orientation in space), and mechanical work was yielded.

Moreover, the total energy of their magnetic fields has increased—and not decreased (as one might surmise). Roughly speaking, the magnetic field of the two combined loops has the same geometric structure as had each of the two loops separately when they were far away from each other, but the strength of the magnetic field at any point in space is now double of what it was before. Then, however, given the quadratic relationship between field strength and energy density, the total energy held by the magnetic field of

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the two combined loops is four times the energy previously held by the field of one single disk alone. Consequently, the total energy held by the two magnetic fields combined has doubled.

This is called Kelvin's principle (since Lord Kelvin was the first who detected it). A good description can be found in Ref. 1 (Chapter C IV, Sec. 52, p. 210):

> "While in mechanics the forces act in such directions that the potential energy is diminished by their action ('work done at the expense of potential energy'), our electrodynamic forces behave in the opposite way: they act in such directions that the energy of the field INCREASES. A particularly clear example of this is the behavior is shown if the currents are maintained at constant strength during the motion, … In this case the energy of the field increases by exactly the same amount of the work done. Thus the double energy gain of amount $2W_{mech}$ per second is balanced by the work performed by the applied e.m.f.s which maintain the constancy of the currents."

This principle was also mentioned by Maxwell[2] (Sec. 638, p. 275) and by Heaviside.[3]

(b) Kelvin's rule makes any integration of $\mathbf{B}^2$ over all space unnecessary but leads nevertheless to exact results. Because of its importance, it seems to be helpful to give a brief derivation of it:

Two equal, current-carrying wire loops (ideal conductors) shall sit far away from each other in empty space to begin with. The energy content of the total magnetic field is then equal to the electric work that was needed in order to establish the two currents in the two wire loops.

That electric work $\mathbf{W_1}$ is

$$\frac{1}{2}I_0\varphi_1 + \frac{1}{2}I_0\varphi_1 = I_0\varphi_1 = \int_{V_{\text{total}}} \frac{1}{2\mu} B_1^2 dV = W_1. \tag{1}$$

Here, $\mathbf{B_1}$ (**x, y, z**) is the magnetic field in the initial situation (when the two wire loops are far away from each other), $\mathbf{I_0}$ is the current strength in each of the two wire loops, and φ_1 is the magnetic flux through the surface area encircled by a wire loop.

As a second step, the two wire loops are brought in contact with each other, so that they sit side by side (with identical orientations in space) as a result. The current strength $\mathbf{I_0}$ in each of the two wire loops is held constant during this process by a power source.

During this displacement, the power source had to deliver the following electric work $\mathbf{W_2}$ (because of an increase in the magnetic flux φ through each of the two wire loops)

$$W_2 = I_0\Delta\varphi + I_0\Delta\varphi = I_0\varphi_1 + I_0\varphi_1 = 2I_0\varphi_1 = 2W_1. \tag{2}$$

The electric work $\mathbf{W_3}$ that would be released if the currents were now reduced to zero is identical in amount to the energy of the total magnetic field. However, given the total magnetic field $\mathbf{B_3}$ is identical in shape with each of the two formerly existing magnetic fields of a single wire current, but has doubled in magnitude (compared with the field of a single wire loop), the energy content of the total magnetic field is four times as large as was the energy content of the magnetic field of a single wire loop during step 1, and is thus twice as large as the energy content of the total magnetic field during step 1

$$W_3 = \frac{1}{2}2I_0\varphi_3 = \frac{1}{2}2I_0 2\varphi_1 = 2I_0\varphi_1 = \int_{V_{\text{total}}} \frac{1}{2\mu}B_3^2 dV = 2\int_{V_{\text{total}}} \frac{1}{2\mu}B_1^2 dV = 2W_1. \tag{3}$$

Therefore, we have

$$W_3 - W_1 = W_1. \tag{4}$$

We find: The increase in energy of the magnetic field during step 2 amounted to $\mathbf{W_1}$, and because the net electric work $\mathbf{W_2}$ invested during step 2 was more than that, namely, 2 $\mathbf{W_1}$, the system must have given off mechanical work by means of the displacement of the two wire loops during step 2. That yield amounted to $\mathbf{W_1}$.

In other words, the increase in energy of the magnetic field during step 2 was as large as the mechanical work given off by the moving wire loops. These two amounts of energy were at the expense of the power source that had to provide electric work during step 2. This is Kelvin's rule.

(c) The principle also applies to permanent magnets that give in to the mutually attractive forces. These can be conceived of as being made up of elementary currents (see below). The currents generate a resulting magnetic field (that holds energy) both outside of the permanently magnetic bodies and in their interior (in empty spaces between the elementary currents).

In other words, given permanent magnets are conceived of as being made up of microscopic, permanent, and steady currents, Kelvin's rule tells us that the mechanical work yielded when two or more permanent magnets attract each other (no matter on which path) is equal in amount to the increase in the energy of the total magnetic field, and it tells us that electric work is provided (by a virtual power source feeding those currents) which is equal in amount to these two gains. No complicated calculations are needed. In Maxwell's[2] words (Sec. 835, p. 473),

> "… the mathematical theory of magnetism is greatly simplified by the adoption of Ampere's theory, and by extending our mathematical vision into the interior of the molecules. In the first place, the two definitions of magnetic force are reduced to one, both becoming the same as that for the space outside the magnet."

But the *real* elementary currents lack of any battery or any other power source. Potential energy of electrons in orbits around an atomic nucleus can be converted into another form of energy, that is, electromagnetic radiation, when electrons are lowering their orbits. But this is undoubtedly not a reservoir that could possibly provide the electric work spent when permanent magnets are approaching each other.

(d) One might raise the question as to whether we made an error by not mentioning potential energy in our energy analysis. The reason for not mentioning potential energy is the following: The energy of the magnetic field (which is being given due consideration) is replacing the role of potential energy. To make this evident, let us consider two positively charged spheres that are sitting close to each other to start with. In that position, each of the two spheres has what is called "potential energy." But this is just another expression for the following: If one holds one sphere at its fixed place and let the other go, that other sphere will be accelerated by the electric field of the stationary sphere, and will pick up kinetic energy. That attainable kinetic energy is the "potential" energy of the sphere. After the second sphere has reached a position far away (where it is still moving), the energy analysis is like this: The kinetic energy which the second sphere is now in possession of is equal in amount to the loss in energy of the electromagnetic field. If one said that, in addition, also "potential energy" was converted into kinetic energy, one would count the same thing twice. This would lead to a wrong result.

II. THE SOLUTION OF A SIMILAR PROBLEM IN RELATIVITY

(a) Where does the energy needed to keep the elementary currents of permanent magnets constant come from? Heaviside[3] had pointed to the fact that a similar problem of an unknown energy source existed in (prerelativistic) gravitation:

> "Now there is a magnetic problem in which we have a kind of similarity of behavior, viz., when currents in material circuits are allowed to attract one another. Now, as Lord Kelvin showed, this double work is accounted for by extra work in the batteries or other sources required to maintain the currents constant. (I have omitted reference to the waste of energy due to electrical resistance, to avoid complications.) In the gravitational case there is a partial analogy, but the matter is all along assumed to be incapable of variation, and not to require any supply of energy to keep it constant. If we asserted that $\mathbf{ce^2/2}$ [equal to $Cg^2/2$ in modern notation] was stored energy [of the gravitational field], then its double would be the work done per unit volume by letting bodies attract from infinity, without any apparent source."

To elucidate, when setting the energy density of the gravitational field as being equal to $\mathbf{1/2\,g^2}$ (times a constant), we have a perfect mathematical analogy with the energy density of the magnetic field, even though there are two magnetic poles (of different sign) rather than just one (as is the case in gravitation). This is because in both cases (gravitational and magnetic), two flat, equal disks can attract each other. While approaching each other, they can yield work and also cause an increase (rather than a *de*crease) in the energy of the total respective field. As Heaviside correctly pointed out, there is a battery (which provides the energy for both the yield of mechanical work and the increase in energy of the field) in the (electro-) magnetic case only; in the gravitational case, there is not.

The same mysterious appearance of energy in the prerelativistic gravitational case was later mentioned by Mie[4] (pp. 34–36):

> "When having two bodies charged with electricity of the same sign approach each other, the energy of the electric state of the ether is increasing; when having two heavy bodies approach each other, the energy of the gravitational field must increase likewise.When having a stone approach the ground, one is gaining energy as work, although the ether, too, is receiving energy in the gravitational field. Where does this energy come from? There is only ONE possible answer to this, and, as far as I know, it was M. Abraham who first found this answer. THE ENERGY EMERGES OUT OF THE HEAVY MASSES THEMSELVES. … When lifting the stone, its elementary particles suck back energy by themselves, and we must provide this energy by delivering work."

[According to Ref. 5 (pp. IV and V), the "ether" is not a substance, but an expression of the recognition that space is not empty; it is the stage for physical phenomena, both of electromagnetic and of quantum-physical nature.]

(b) The energetic problem arises in General Relativity as well (and not only in *prerelativistic* gravitation), as it can be shown that the gravitational field carries no energy at all [see Refs. 6 and 7; of course, one must not confuse energy of the gravitational field with the gravitational potential energy (of a test mass of unit size); the latter is *not* zero]. Therefore, the gravitational field cannot be a source of any flow of gravitational energy. This is also what Misner *et al.*[8] are postulating in their famous standard textbook on gravitation (Chap. 20.4 "Why the energy of the gravitational field cannot be localized," p. 467):

> "Moreover, 'local gravitational energy-momentum' has no weight. It does not curve space. It does not serve as a source term on the righthand side of Einstein's field equations."

Given this paper is not about the energy of the gravitational field, not much effort can be put into showing that the gravitational field carries no energy (see the references above, instead). Just one simple reflection shall be added: If the energy density of the gravitational field were not zero and were proportional to the negative square of the gravitational acceleration $\mathbf{g}$ (as is assumed in some textbooks), one would run into a new dilemma: A hollow sphere of heavy mass has a zero **g**-field in its interior. Nevertheless, if the hollow sphere is contracting, an enormous amount of kinetic energy can be generated (which would amount to infinity if the radius of the sphere eventually approached zero). This phenomenon is called Kelvin–Helmholtz contraction. But then the formerly field-free space must have contained an enormous, if not infinite amount of energy. However, this is incompatible with the assumption that the energy density of the gravitational field is proportional to the negative square of **g**.

In case the energy of the gravitational field were proportional to the positive (and not the negative) square of **g**, we would run into the dilemma presented by Heaviside.

(c) As regards General Relativity, the energy dilemma is solved by the recognition that Einstein's field equation and the principle of local conservation of energy, when combined, require the existence of a hidden reservoir of energy that is located nearby, but in the direction of a fourth spatial dimension.[6,7,9]

That same hidden reservoir of energy seems to be the source of the energy flow into the magnetic field of the two disks (which are approaching each other), as will be shown below.

III. ANALYSIS OF THE POYNTING VECTOR FIELD

A. Model magnets with a capacitor as a power source

1. Sinks

(aaa) Let us imagine that we have built two model magnets that are to mimic two permanently magnetized bodies. The two bodies which are to be mimicked shall have the shape of two disks. The elementary currents shall be replaced by a large number of small, loop-shaped wire currents held constant by a smart power supply. This power supply shall be a parallel-plate capacitor. It shall be connected to the loop-shaped wire circuits by several coaxial cables. In the reference frame of the lab, both (mimicked) permanent magnets shall be approaching each other at equal but opposite speeds. Then the pointing vector field, that is

$$\mathbf{S}(x,y,z) = \varepsilon_0\, c^2\, (\mathbf{E}\times\mathbf{B}), \tag{5}$$

makes the following flows of electromagnetic energy visible (when the two disk-shaped model magnets are approaching each other in the reference frame of the lab; **S** is the intensity of the flow of electromagnetic energy in Joule per second and per m^2, **E** is the electric field, **B** is the magnetic field, **epsilon$_0$** is the dielectric constant in vacuum, **c** is the speed of light in vacuum): The capacitor (power supply) is a *source* of a flow of electromagnetic energy (see below), and both the small, loop-shaped, current-carrying wires and the regions in space where the energy density of the magnetic field is increasing are *sinks* (see below).

Although the current-carrying wires are sinks, the internal energy of the wires does not change (if we assume that the wires are perfect conductors). This is because they are giving off mechanical work—in the reference frame of the lab—at the same rate at which they are receiving electromagnetic energy while they are acting as sinks, similar to armatures of electric motors.

(bbb) In order to realize all this, we consider the criteria for sources and sinks of flows of electromagnetic energy in a general form. For a region of space or an object in space to be a *sink*, the following criterion must be met (see Ref. 5, Sec. 299, p. 403; **W** is density of energy of any kind in three-dimensional space in Joule per m^3, **t** is time, **j** is the electric current density in Amp/m^2, and $\boldsymbol{\mu_0}$ is the permeability of the vacuum):

$$-\nabla\cdot\varepsilon_0\mathbf{c}^2(\mathbf{E}\times\mathbf{B}) = -\left[\varepsilon_0 c^2(\nabla\times\mathbf{E})\cdot\mathbf{B} - \varepsilon_0\boldsymbol{c}^2(\nabla\times\mathbf{B})\cdot\mathbf{E}\right] = \frac{\delta W}{\delta t} = \frac{\delta}{\delta t}\left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu}B^2\right) + \mathbf{j}\cdot\mathbf{E} > 0. \tag{6}$$

The very left-hand side of Eq. (6) represents nothing but the (negative) divergence of the Poynting vector as presented in Eq. (5). A general principle of vector calculus is applied thereby (see Ref. 10, Sec. 5, Eqs. 2, 2a, and 2b, p. 24/25, for a derivation of that principle).

The second summand in the square bracket in Eq. (6) is irrelevant in empty space. This is because the curl of the vector **B** is zero in empty space (if neglecting an electromagnetic wave that is generated by the motion of the magnets). Hence, when considering empty space inside (in empty space between the wires) or outside the moving model magnets, only the *first* summand in the square bracket is relevant (whereas the second summand is vanishing).

As the net energy contained in the total magnetic field is increasing, it follows that there must be regions in empty space which act as sinks. In other words: There must be regions in empty space in which the first summand in the square bracket of Eq. (6) is not vanishing, and is larger (and not smaller) than zero.

(ccc) What is the contribution of the *second* summand in the square bracket to the overall result? As stated above, that summand is irrelevant in empty space. Only in the interior of a current-carrying wire (and in the gap between the capacitor plates, see below) is it that the curl of the vector **B** is different from zero. That is to say: The second summand in the square bracket of Eq. (6) can only be relevant when it comes to scrutinizing the interior of current-carrying wires (or the interior of a capacitor). For the current-carrying wire to act as a sink [that is, in order for the second summand in the square bracket of Eq. (6) to be non- vanishing], there must be an electric field **E** along the wire (as required by the dot product of the curl of **B** and the electric field **E**). But if the wire material is a perfect conductor, no electric field can exist in it. The divergenceless electric field, which, in the reference frame of the lab, is generated as a result of the change in the magnetic field at a fixed place, is, in the interior of the wire material, neutralized by the electrostatic field of the power source (capacitor). (One should note that both the change in the magnetic field at a fixed place and also the divergenceless electric field are brought about by the displacement of the model magnets, and not by a switching some electric currents on and off. The divergenceless electric field is therefore described by the Lorentz transformation of electric and magnetic fields.)

Hence, the only way for a net electric field in the wire to exist is by the "catalytic" action of a Lorentz force: The Lorentz force is trying to stop the electric current; as a consequence, the battery (the capacitor, respectively), whose task is to maintain a constant current strength by increasing the voltage, if necessary, builds up an electrostatic field along the wire that counteracts the Lorentz force and neutralizes its force-effect on charges. Since the Lorentz force is no electric field, a net electric field is present along the wire, even though its effect on movable charges is (almost completely) neutralized by the Lorentz force. This is why the dot product of the electric field **E** and the curl of **B** can be different from zero, so that the current-carrying wire acts as a sink (like the armature of an electric motor).

(ddd) One should note that, in the reference frame of the *wire*, a Lorentz force that would act on the wires (which constitute elementary model magnets) does not exist. It is replaced by a relativistic electric field generated by the motion of the magnet. As has been mentioned already, the relativistic electric field is described by the Lorentz transformation of electric and magnetic fields. This relativistic electric field neutralizes the electrostatic field of the power supply, so that there is no resulting electric field along the wire in the reference frame of the wire, even though an electric current is present along the wire.

The relativistic electric field as it presents itself in the reference frame of the wire is thus stronger than the relativistic electric field as it presents itself in the reference frame of the lab. This is easily accounted for by the fact that the velocity of the source of the magnetic field, i.e., the other permanent magnet, is twice as high as what it is in the reference frame of the lab. However, in the reference frame of the lab, the relativistic electric field is supported—in its force-effect on electrons—by a Lorentz force (which, in turn, does not exist in the reference frame of the wire). See Ref. 11 (Chapter 7.2, p. 262) on an analogous situation in which a conducting rod is moving through a magnetic field whose source is stationary in the reference frame F of the lab, but is in motion in the reference frame F' of the rod:

> "An observer in F says: 'Inside the rod there has developed an electric field E [caused by induced charges on the surface of the rod]… exerting a force qE … which just balances the force q **v × B** [Lorentz force] that would otherwise cause any charge q to move along the rod.' An observer in F' says: 'Inside the rod there is no electric field [the electrostatic field of the induced surface charges and the relativistic electric field (described by the electromagnetic Lorentz transformation) generated by the motion of the magnet just cancel each other], and although there is a uniform magnetic field here, no force arises from it because no charges are moving.' Each account is correct."

Since the Lorentz force thus only exists in the reference frame of the lab (where the wire is in motion), it is only in that frame of reference that the current-carrying wire acts as a sink. The current-carrying wire is not a sink in its own frame of reference. Hence, flows of electromagnetic energy are completely frame-dependent as regards their existence.

[One realizes that the often used "flux rule" is not always accurate enough: According to that rule, the electric voltage induced in a piece of wire (in motion across a magnetic field) is proportional to the magnetic flux that is "cut" by the wire per second. In order to apply the "flux rule," it is not necessary to distinguish between a relativistic electric field generated by a moving magnet acting on a charge and a Lorentz force which turns up when a charge is being moved through a magnetic field. So far, so good. But for the Poynting vector, it is essential to distinguish as to whether the voltage is generated by a Lorentz force, or by a relativistic electric field, instead. A Lorentz, which acts on a wire, is present solely in a reference frame in which the wire is in motion. Although the Lorentz force can generate a voltage along the piece of wire, that voltage is not the result of an electric field **E**, from which the Lorentz force must be distinguished.]

2. Sources

As regards regions or objects that want to qualify as a *source* of a flow of electromagnetic energy, the general criterion is

$$-\nabla\cdot\varepsilon_0\mathbf{c}^2(\mathbf{E}\times\mathbf{B}) = \left[\varepsilon_0 c^2(\nabla\times\mathbf{E})\cdot\mathbf{B} - \varepsilon_0 c^2(\nabla\times\mathbf{B})\cdot\mathbf{E}\right] = \frac{\delta W}{\delta t} = \frac{\delta}{\delta t}\left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu}B^2\right) + \boldsymbol{j}\cdot\mathbf{E} < 0. \tag{7}$$

In the case of our model magnets (equipped with a power supply), the current-carrying wires are clearly no sources, as we have identified them as sinks already.

For reasons of simplicity, we assume that the power supply is a parallel-plate-capacitor which sits far away from the two model magnets. The discharging of the capacitor generates a displacement current **dE/dt** between its parallel plates. This displacement current, in turn, generates a magnetic field **B** whose curl is different from zero between the capacitor plates. In combination with the electric field **E** between the plates, the discharging capacitor thus acts as a source of a flow of electromagnetic energy [see Ref. 5, Sec. 299, Fig. 194, p. 403; see also Ref. 12 (1907), Sec. 86, pp. 375–377]. More precisely: The second summand in the square bracket of Eq. (7), that is, the dot product of the vector **E** and the curl of the vector **B**, is different from zero between the capacitor plates. The (partial) differential quotient **dW/dt** is thus negative inside the parallel-plate capacitor. More precisely: The local reservoir of energy which sits there is losing energy. It is losing energy because $\mathbf{E^2}$ is declining. Since the discharging current of the capacitor is declining in strength with time, $\mathbf{B^2}$, too, is declining with time. The dot product of **J** and **E** (which also appears in the equation as a summand that contributes to the local energy density) is zero here, as the electric field between the capacitor plates does no work on moving charges, nor are moving charges doing work against that electric field—there are simply no moving charges between the capacitor plates.

All changes shall be performed so slowly that the energy of an electromagnetic wave that is radiated away can be neglected. The energy radiated away by an accelerating charge amounts to [see Ref. 11, Appendix B, Eq. (5), p. 462]

$$W = \frac{1}{4\pi\varepsilon_0}\frac{2}{3}\frac{q^2a^2t}{c^3} = \frac{1}{4\pi\varepsilon_0}\frac{2}{3}\frac{q^2\sqrt{2s}\,a^{3/2}}{c^3}. \quad (8)$$

It vanishes whenever the acceleration "**a**" and the distance "**s**" (over which the accelerating force is acting) are small enough.

Several coaxial wires shall connect the power supply (capacitor) with the two model magnets and their numerous wire loops. The current in a coaxial wire does not generate a magnetic field in the exterior. The magnetic field is confined to its interior, instead. The flow of electromagnetic energy that leaves the capacitor enters the interior of the (parallel) coaxial cables, and, due to a lack of an appropriate electric field in the vicinity of the capacitor, goes nowhere else.

It should be mentioned that some regions of empty space, too, act as sources. This is because not all regions of space in which a magnet is in motion experience an increase in density of the magnetic field during the displacement of the magnets. Some regions experience a decrease, instead [see Ref. 5, Sec. 300, Fig. 195, p. 404, for a depiction of the energy flow around a single electrically charged sphere that is being displaced]. The latter regions function as a source, and the first summand of Eq. (7) is non-vanishing and smaller than zero right there.

3. Energy balance of sources and sinks

Given the speed of the flow of electromagnetic energy is as high as the speed of light and therefore infinite in a practical sense, the total amount of electromagnetic energy picked up by all sources must, at any moment, equal the amount of electromagnetic energy delivered in all sinks.

But even more: The principle of local conservation of energy requires that the delivery of electromagnetic energy at a sink leads to a pile up of energy in any form right there. This is why **dW/dt** has to be numerically positive, and it has to be equal in magnitude to the performance of the sink (in Joule per second and per m^3). At a source, it is just the other way round: Here the principle of local energy conservation requires that the outflow of electromagnetic energy leads to a diminishing of energy in any form. This is why **dW/dt** has to be numerically negative, and it has to be equal in magnitude to the performance of the source (in Joule per second and per m^3).

According to Kelvin's principle, the amount by which the energy stored in the capacitor (as a power supply) is diminishing per second is equal to the amount by which the energy of the total magnetic field is increasing per second, plus the total mechanical work that is being given off per second (with the two summands being equal in magnitude).

B. Model magnets with a mechanical power supply (similar to the magnet generated in Rowland's experiment)

1. Sinks and sources

Let us now remove the capacitor as a power supply. Instead, we imagine that each of the many elementary currents of the model magnets is produced mechanically by making a small nonconducting round disk, loaded with fixed negative electric charge, spin about its axis of symmetry. This shall be achieved in the same way as in Rowlands famous experiment of 1875. More precisely: We imagine that the rim of a small disk (of which there are many) is permanently soaked with fixed electric charge of the same negative sign. The fixed negative charge shall not be confined to the surface of the plate, but can be found in the interior of the material as well, though only a short distance from the rim. (For reasons of symmetry, the resulting electrostatic field generated by the fixed electric charge particles has no component in a tangential direction.) When a small disk rotates, it represents a single elementary current (of which there are many). In order to keep the model magnets electrically neutral, positive charges are fixed to other places in the interior.

Both the divergenceless, relativistic electric field (generated by the change in the magnetic field) and also the Lorentz force (see above), both of which are present in the reference frame of the lab, obstruct the (presupposed) constant spinning motion of every small disk. For a disk to keep spinning, a mechanical force must overcome these obstructions. In doing so, this mechanical force does work (which is absorbed by the small disk).

Different from the previous version of the model magnets, the component of the divergenceless, relativistic electric field (as it presents itself in the reference frame of the lab) which obstructs the motions of the negative charge carriers is no longer neutralized by an electrostatic field of a power supply. This is why it enters the second summand in the square bracket of Eq. (3) as a nonvanishing **E**. Also different from the previous variant of the model, that field **E** is directed against the motion of electric charge (given it is obstructing the motion of charged particles, that is, the spinning of the small disk) and is no longer going along with it.

Since the curl of **B** in the interior of a small disk is thus nonvanishing, the rim region of a spinning small disk now acts as a source (and not as a sink) of a flow of electromagnetic energy, similar to the armature of an electric generator.

2. Energy balance of sources and sinks

We realize that the role of the capacitor (and the energy stored in it) is now being played by the reservoir of the mechanical work which is absorbed by the small spinning disks (representing elementary currents) in order to keep them spinning at a constant rate despite the obstructive action of the relativistic electric field and also the despite the obstructive action of the Lorentz force (which turns up in the reference frame of the lab where it is not only the source of the magnetic field but also the charge of the disks that is in motion).

According to Kelvin's rule, half of the mechanical work spent on the many small disks per second is converted into mechanical work delivered to the ambient during the linear displacement of the model magnets. The other half is converted into a flow of electromagnetic energy that leaves the rim regions of the many small disks (which act as sources) in order to increase the energy of the magnetic field in some regions of empty space (which act as sinks).

C. Real (permanent) magnets, the energetic significance of unstoppable electron spins, and Mie's correct assessment regarding energy flows into particles and out of them

1. Application of the Poynting vector (as an instrument of showing energy flows) on microscopic currents as justified by the Einstein-de-Haas effect

When it comes to real magnets, the Einstein-de-Haas effect tells us that magnetism in iron is caused mainly by spinning or orbiting particles that have angular momentum and hence mass as well as a magnetic moment. Although the angular momentum of each elementary current turned out to be only half in magnitude of what it had been supposed to be (just as if some positive charge took part in constituting the current by spinning in the opposite sense), that feature does not deprive our model of its character as a qualitatively valid description of nature. In other words: The Einstein-de-Haas effect justifies the assertion that our model magnets (mimicking permanent magnets) are reliable copies of nature with respect to flows of energy. As Purcell[11] (Chapter 11.6, p. 419) expressed it:

> "We need not even go so far as to say IT IS a current loop. What matters is only that it behaves like one in the following respects: (1) it produces a magnetic field which, at a distance, is that of a magnetic dipole; (2) in an external field B it experiences a torque equal to that which would act on a current loop of equivalent dipole moment; (3) within the space occupied by the electron, div B = 0 everywhere, as in the ordinary sources of magnetic field with which we are already familiar."

The last part of Purcell's statement is especially important: Given **div B = 0** within the space occupied by the spinning electron (as it would be the case when dealing with an ordinary current loop), there is also such a thing as a well-defined **curl of B** in the space occupied by the spinning electron. That **curl of B** is not zero everywhere. Therefore, we are justified in applying the Poynting vector as an instrument of detecting flows of electromagnetic energy in space also when it comes to the elementary currents that make up real, permanent magnets.

We find that the second summand in the square bracket of Eq. (2) is different from zero within the space occupied by a spinning electron.

But we neither find a capacitor nor a visible reservoir of mechanical work as a power supply. Nevertheless, the flows of electromagnetic energy—made visible by the Poynting vector—are still the same as they were in the latter version of our model magnets where we had a power supply (capacitor). That is to say: When the two permanently magnetic disks approach each other, the Poynting vector shows flows of electromagnetic energy coming out of the elementary currents, that is, coming out of the spinning electrons of the magnetic material.

When drawing a picture of the flow of electromagnetic energy in the vicinity of a spinning electron, we would have to guess the electron's exact location at a given moment in time because of the uncertainty principle of quantum mechanics. But this does not pose an obstacle to identifying the electron qualitatively as a source of a flow of electromagnetic energy in the picture provided to us by the Poynting vector.

In other words, in the real case, there is no such thing as a visible energy reservoir (feeding the source of the flow of electromagnetic energy) whose contents would everywhere be *diminished to the appropriate extent*. We rather have (at some places in empty space)

$$\begin{aligned} &\left|-\left[\varepsilon_0 \boldsymbol{c}^2(\nabla\times\mathbf{E})\cdot\mathbf{B}-\varepsilon_0 \boldsymbol{c}^2(\nabla\times\mathbf{B})\cdot\mathbf{E}\right]\right| \\ &=\left|\left[\varepsilon_0 \boldsymbol{c}^2(\nabla\times\mathbf{E})\cdot\mathbf{B}\right]\right| \neq \left|\frac{dW}{dt}\right| \\ &=\left|\frac{\delta}{\delta t}\left(\frac{1}{2\mu}B^2+\frac{\varepsilon_0}{2}E^2\right)+\mathbf{j}\cdot\mathbf{E}\right|. \end{aligned} \tag{9}$$

2. The anticipation of a recognition of this kind by Mie

Mie[4] (pp. 34–36) anticipated such a recognition, though only vaguely, by switching from gravitation (see above) to electromagnetism:

> "We may even say that it is more difficult to understand why one is not observing a similar influence of the state of the ether also on the energy nodes [charged particles] contained in it when it comes to electric and magnetic fields. We will have to assume that an influence is present also in this case, but we still don't know anything for sure about it yet."

Hence, Mie (correctly) assumed that the energy which emerged in the three-dimensional world when permanent magnets were approaching one another "came out of the energy nodes themselves," and was "sucked in" by these nodes when the magnets were separated from each other.

D. The postulate of a fourth spatial dimension as a consequence of the picture provided by the Poynting vector and the principle of action by contact combined

1. The principle of "action by contact" in general

The Poynting vector is a reliable instrument for detecting any sources of a flow of electromagnetic energy. Consequently, in cases in which the Poynting vector shows a *sink* (of a flow of electromagnetic energy) where energy is piled up, but no *source* at which the contents of any energy reservoir is diminished, the nevertheless existing, tapped energy reservoir cannot be a reservoir which is located in three-dimensional space. In particular, there is no room for assuming that the (extensionless) electron stores energy in its interior. If it could store energy in its interior, its rest mass could not be constant. Moreover, given there is no upper limit to the strength of the relativistic electric field generated by a displacement of magnets (as the fields of those magnets can be as strong as one wishes them to be), there is no upper limit to the amount of energy given off by an elementary current during a single displacement. In order to save the principle of local conservation of energy and the principle of action by contact, one must thus conclude that the tapped energy reservoir sits in the direction of a fourth spatial dimension.

The necessity for this conclusion (of the existence of a fourth spatial dimension) is rooted in the principle of "action by contact" and the (related) principle of conservation of energy, which is understood as a principle of *local* conservation of energy. Planck[13] (Introduction, Sec. 1, p. 1/2) expressed the local principle of "action by contact" in the following way:

> "According to this principle, there cannot exist immediate causal effects into distance; i.e., it cannot occur that the effect of a local event suddenly surfaces at a more or less distant place with skipping the objects that sit in between. Every causal effect is rather spreading across space from point to point at finite speed For according to the latter [the theory of action by contact], when calculating effect sizes, one doesn't need to care about what is happening at other, finitely distant places, but may restrict oneself to considering events in the immediate neighborhood, whereas, if adopting actions at a distance, one is, strictly speaking, obliged to search the whole universe for places from which the effects that are to be calculated could be influenced directly in a noticeable manner."

One thus has to realize that the spin of electrons—which is mainly responsible for magnetism in permanent magnets—acts as a gateway to an energy reservoir that sits in the fourth spatial dimension. In the course of decades, physicists have got used to the astonishing fact that the spin of electrons, revealed for the first time in the famous Stern–Gerlach experiment of 1922, is unstoppable. But physicists have not yet realized that this strange behavior of unstoppability entails the existence of a hidden reservoir of energy in a fourth spatial dimension, given the principle of local conservation of energy and the principle of action by contact are physically correct. This nearby reservoir of energy might even be responsible for some so far unexplainable phenomena like ball-lightning: From time to time, energy from the hidden reservoir that is sitting nearby in the fourth spatial dimension might emerge in three-dimensional space by random fluctuations.

2. Topological consequences (fourth spatial dimension, brane-unverse) of the principle of "action by contact"

If one adheres to the two principles just mentioned, the question of how many spatial dimensions exist in physical space becomes an empirical one. Reichenbach[14] (Sec. 12, p. 80 and Sec. 44, p. 275) made this very clear:

> "Topology is an empirical matter as soon as we introduce the requirement that no causal relations must be violated, causal effects cannot reach distant points of space without having passed through the intermediate points. Though the above definition of 'between' the principle of action by contact becomes the more fundamental principle of spatial order; the neighborhood relations of space are to be chosen in such a way that the principle of action by contact is satisfied. This principle expresses the prescription which our concept of causality yields for the topology of space. This rule determines the dimensionality of space …."

In exactly this manner, the existence of a hidden energy sink in the fourth spatial dimension was attempted to be proved empirically in the Fermi National Accelerator Laboratory (by a team led by Landsberg of Brown University) and in DESY's electron–proton collider in Germany. See Ref. 15:

> "Collision experiments carefully reconstruct all particles emerging from a collision. A possible sign of extra dimensions would be a collision in which a particle—and hence energy—'disappeared,' perhaps indicating a graviton leaving our visible universe and entering extra spatial dimensions—the megaverse."

However, there is no need for particle collision experiments in this context. A necessity for assuming a fourth spatial dimension can be arrived at by simple analytical reflections on Maxwell's equations and their consequences for permanent magnets.

It is commonly agreed that, in case space has four dimensions, all objects in the world must be spatially extended in all four dimensions and not in just three. This must apply to ourselves and things around us as well. But as we do not perceive ourselves as being extended in all four spatial dimensions, we have to conclude that the extension of material objects like ourselves or planet Earth in the fourth spatial dimension is only microscopical. Our world thus constitutes a spatially four-dimensional, extremely thin "brane," as it was described by Randall.[16]

IV. EVIDENCE OF AN ENERGY RESERVOIR HIDDEN IN A FOURTH SPATIAL DIMENSION PRESENTED BY THE LAGRANGIAN FORM OF MECHANICS

The Lagrange-function for a charged particle in an electric and in a magnetic field reads [see Ref. 17, Lecture 11, Eq. (14), p. 201]

$$d\int_1^2 L\,dt = d\int_1^2 (T-U)\,dt$$
$$= d\int_1^2 \left[\frac{m}{2}\mathbf{v}\cdot\mathbf{v}+\frac{q}{c}\mathbf{v}\cdot\mathbf{A}(x,y,z)-qV(x,y,z)\right]dt = 0. \tag{10}$$

Here, **L** is the difference between the kinetic energy of the charged particle (whose charge is **q**) and its potential energy **U** in the electric field. The vector **v** is the velocity of the particle, and **A** is the vector potential at its momentary location. The dot product of **v** and **A** (multiplied by a constant) is a specially introduced term that allows the derivation of the Lorentz force from that equation. It must be considered as a form of energy—similar but different to both the kinetic energy of a charged particle and its potential energy in the electric field.

The vector potential is the vector whose curl is equal to the magnetic field **B**. It is called "potential" for the following reason: In case there exist spatially fixed electric currents in space that are flowing in the **x**-direction, the electric current density in the **x**-direction is treated as if it were an electric charge density. The electric potential (as determined by Coulomb's law) generated by that imagined distribution of charge in space gives the **x**- component of the vector potential **A**. The other components are obtained in an analogous manner.

The physical reality of the vector potential **A** can hardly be doubted. As Ref. 17 (Lecture 11, p. 197) formulated it:

> "… without it, we could not express the principle of stationary action, or the Lagrangian, Hamiltonian, and Poissons formulation of mechanics for particles in magnetic fields."

Let us now consider a toroidal coil in which the alternating electric current has reached a maximum in strength ($\mathbf{dI}/\mathbf{dt} = 0$) at the moment considered. With the electric current in the wire being constant for a while, there is neither a magnetic field nor an electric field outside of the toroidal coil, even though the vector potential outside of the coil has not vanished, but is at a maximum. Despite its being there, it has no mass and no weight, given the energy (=mass) density of electromagnetic fields is composed of the squared electric and the squared magnetic fields, but does not contain the vector potential [see Eq. (2)].

When the vector potential at a given location is changing with time, an inductive electric field turns up

$$\mathbf{E}_{\text{ind}} = -\frac{\delta \mathbf{A}}{\delta t}. \tag{11}$$

When the curl of the vector potential **A** is not vanishing (which happens inside the coil), a magnetic field **B** turns up

$$\nabla \times \mathbf{A} = \mathbf{B}. \tag{12}$$

Hence, the vector potential **A**, although, under certain circumstances, being existent in space all the time, takes on mass and weight only when it is changing with time or when its curl is different from zero. This suggests a connection of the vector potential with a reservoir of energy that sits nearby, but in a fourth spatial dimension.

V. INERT MASS AND HENCE ENERGY HIDDEN IN MORE-DIMENSIONAL SPACE AS A CONSEQUENCE OF THE PRINCIPLE OF "FORCE AND COUNTER-FORCE" IN ELECTROMAGNETISM

A. Force on a magnet in an external field, and the search for the object on which the counterforce is exerted

A confirmation of a hidden mass or energy sitting in a fourth spatial dimension is achieved by the following reflection: Once again, imagine that two equal, permanent magnets separated from each other over some distance in empty space are attracting each other. They shall not be in motion with respect to one another, but shall be at rest.

Even if we assume that the two magnets are subject to equal but opposite forces, we cannot yet say that the principle of "force and counterforce" or "action and reaction" (action being force times time) are surely observed. This is because those two principles are understood as local ones (action by contact), and not as principles that would work on the basis of "action at a distance" (see above). Einstein[18] (Chapter XIX, p. 63) explicitly mentioned such a situation in the following way:

> "As a result of the more careful study of electromagnetic phenomena, we have come to regard action at a distance as a process impossible without the intervention of some intermediary medium. If, for instance, a magnet attracts a piece of iron, we cannot be content to regard this as meaning that the magnet acts directly on the iron through the intermediate empty space, but we are constrained to imagine – after the manner of Faraday – that the magnet always calls into being something physically real in the space around it, that something being what we call a 'magnetic field.' In its turn this magnetic field operates on the piece of iron, so that the latter strives to move towards the magnet."

This is made evident when imagining that the source of the magnetic field which operates on Einstein's piece is iron is suddenly removed. The magnetic field around the piece of iron and hence the magnetic force on the piece of iron will nevertheless be left unaffected by this for a while, since the causal front (which carries the news of a disappearance of the field's source) needs time to reach the piece of iron. During this short period of time, it is particularly obvious that Einstein's piece of iron, which is still feeling an unchanged magnetic force in the same direction as before, must push itself away from some inert object in the opposite direction in order to obey the rules of "force and counterforce" and "action and reaction."

[In the face of this apparent jeopardy to the principle of force and counterforce (or action and reaction) and even to the principle of conservation of energy, Abraham[19] (p. 10) proposed the following solution for cases in which two bodies exerting forces on each other are spatially separated, and in which said principle might therefore be in trouble:

> "One has to consider the possibility that force, just like energy, is in a latent state for a while."

Such a latent state would allow forces and energies to emerge out of nothingness, and to vanish into nothingness. It would be a departure from basic principles of physics, which, however, is unnecessary und not demanded by experience.]

However, when the magnetic field generated by first magnet "operates" on the second magnet (or, when the magnetic field mentioned by Einstein operates on the piece of iron), the principle of "force and counterforce" or "action and reaction" would (apparently) only be observed if a counter momentum (force times time) had been transferred to the *magnetic field* (surrounding the second magnet or Einstein's piece of iron). This is because the inert mass of the magnetic field appears to be the only object in contact with the piece of iron or the second magnet onto which a counter momentum could have possibly been transferred. Simply speaking, apparently, the second magnet (or Einstein's piece of iron) must have been pushing itself away from the inert mass of the ambient magnetic field in a direction opposite to the magnetic force that it has been feeling.

But given the inert mass of the magnets is chosen to be extremely large, the magnets do not pick up any noticeable speed despite being subject to an accelerating force, and the change in the magnetic field with time (viewed in the reference frame of a lab) at any place would thus approach zero. As a consequence, there would be no electric field, but only the magnetic field. This, in turn, leads to the conclusion that the finitely large energy (=mass) of the magnetic field around the second magnet is not in motion (even after a longer period of time), so that no momentum can have been transferred to it. We hereby assume that the first magnet is not moving at all, so that, in the reference frame of the lab, all the change in the resulting magnetic field occurring in the immediate vicinity of the second magnet is brought about by a displacement of the second magnet and its magnetic field. But this displacement is negligible if the mass of the second magnet is large enough. Wien[20] (pp. 714/715) gave the following description of such a situation:

> "From the equations … we conclude that energy is at rest if there exist either exclusively electric or exclusively magnetic lines of force in the system."

In other words: The term **dB/dt (x,y,z,t)** depends on the mass of the second magnet. At a given magnetic field generated by the second magnet, the absolute magnitude of its change with time is smaller if the mass of the second magnet is larger. Then, however, the strength of the generated electric field **E** (which is a function of **x,y,z** and of time **t**), and thus the increase in momentum density of the flow of electromagnetic energy (the momentum density being equal to $\mathbf{S}/\mathbf{c}^2$ and hence proportional to the cross product of the vectors **B** and **E**) around the second magnet depend on the mass of the second magnet as well. But the increase in momentum of the second magnet itself (force times time)—brought about by the magnetic field generated by the (stationary) first magnet—does *not* depend on the mass of the second magnet.

Therefore, the inert mass of the magnetic field does not qualify as an object onto which a sufficiently large counter momentum could be transferred. In order for the principles of "force and counterforce" and "action and reaction" to be observed, some hidden inert mass (=energy) must exist onto which the counter momentum (or at least a fraction of it) is transferred.

The counter momentum is produced in one of the three familiar directions of space (as required by the principle of action and reaction), although it is, quite obviously, parallelly displaced in a fourth spatial direction. This is why it cannot be observed in three-dimensional space. Simply speaking: The second magnet (or Einstein's piece of iron) is pushing itself away from some invisible inert mass. That mass is invisible, because it exists—nearby—in the direction of a fourth spatial dimension. The counter momentum which it is receiving is hence parallelly displaced in this fourth spatial dimension.

B. Electric force on a plate of a charged parallel-plate capacitor, and the search for the object on which the counterforce is exerted

Another reflection leads to the same result. Imagine an electrically charged parallel-plate capacitor with round plates. The distance between the two plates shall be small compared to the diameter of a plate. Each of the two plates is subject to an electric force caused by the electrostatic field of the other plate. Let us assume that the mass of each plate is not extraordinarily large, so that the plates we are set in noticeable motion toward the center of the gap (between the plates).

On which object—in contact with the plates—is the counterforce acting? One might think it is the mass of the homogeneous electrostatic field between the plates toward which the capacitor plates are pulling itself. But when the gap between the two plates is narrowing, the homogeneous structure and the strength of the electrostatic field does practically not change (except for the fringe region). Therefore, no magnetic field is being generated there. Then, however, the Poynting vector is zero—and stays zero—between the plates, and no change in momentum of the energy of the electromagnetic field is brought about when the plate is moving.

There is, though, a weak magnetic field generated by the motion of the electrically charged capacitor plates. This magnetic field encircles the electrostatic field lines that extend between the two plates. The combination of this magnetic field and the electrostatic field between the plates is responsible for the existence of a weak Poynting vector field that reveals a flow of electromagnetic energy from the vanishing fringe region of the plates (where the electrostatic field is not homogeneous and extends somewhat into surrounding space) into the interior of the capacitor. But the momentum of that weak flow of electromagnetic energy is directed at right angle to the electric force experienced by the moving capacitor plate. This is why this electromagnetic energy could not function as an object that is receiving a counterforce, even if the increase in momentum of the transversal flow were much stronger than it actually is.

Again, the assumption of a hidden inert mass is indispensable.

VI. INERT MASS (AND HENCE ENERGY) HIDDEN IN HIGHER-DIMENSIONAL SPACE AS A CONSEQUENCE OF THE PRINCIPLE OF ACTION BY CONTACT IN RELATIVISTIC MECHANICS

In 1916, Einstein imagined two fluid bodies of the same mass floating in empty space some distance away from each other and from all other masses. The mutual distance of the two fluid bodies from each other shall be invariant. In the reference frame of an observer who does not feel any external force that would act on himself or herself, one of the two fluid bodies shall be spinning around an axis which is identical with the connecting line between the two bodies. Consequently, that body is no longer spherical in shape, but is an ellipsoid.

What is the reason for this difference between the two bodies? The usual answer is: The latter body is spinning relative to the distant stars and galaxies, and this is thought to be the cause for its nonspherical shape. Einstein[21] [Sec. 2 (The need for an extension of the postulate of relativity), p. 113] agreed:

> "The only satisfactory answer must be that the physical system consisting of S_1 and S_2 reveals within itself no imaginary cause to which the differing behavior of S_1 and S_2 can be referred. The cause must therefore lie outside this system The mechanical behavior of S_1 and S_2 is partly conditioned, in quite essential respects, by distant masses which we have not included in the system under consideration. These distant masses and their motions relative to S_1 and S_2 must then be regarded as the seat of the causes of the different behavior of our two bodies S_1 and S_2."

However, this statement, which has been called Mach's principle, is incompatible with the principle of action by contact, as Mach's principle postulates nothing else but an action at a distance (which Einstein is rejecting so vehemently in the electromagnetic context, see above). For the information on the motions of the distant masses is not contained in a local gravitational or electromagnetic field.

Apart from violating the principle of action by contact, this form of Mach's principle is not supported by General Relativity. This is because universes can be conceived of in which the appearance of centrifugal forces in a small, spinning body cannot be caused by a rotation relative to the distant masses of the universe. As Rothman[22] (p. 344) puts it:

> "The key difference was that Gödel's universe rotated, meaning that distant galaxies rotate with respect to a gyroscope sitting on my desk, and that anyone, anywhere in the universe would observe the same behavior. (It does not mean that the universe is rotating around some central axis.) For true followers of Mach, a gyro should track the bulk matter of the cosmos, and so it should remain stationary with respect to distant galaxies. Since Gödel, researchers have found other rotating models of the universe, all of which similarly contradict Mach's premise. Such models can be declared unphysical, however, because they flagrantly contradict observations of the real universe. Nevertheless, as theoretical solutions they demonstrate the difficulties that come with defining inertia purely in relation to other objects."

What is needed when adhering to the principle of action by contact is a *local* cause. The local cause can only be an inert, *nearby* mass, relative to which one of the two fluid bodies is spinning. The only local mass which is available is the mass of the nearby reservoir of energy hidden in the fourth spatial dimension.

VII. THE RELATEDNESS OF ELECTROMAGNETIC AND GRAVITATIONAL FLOWS OF ENERGY; CYCLICAL PROCESSES WHICH COULD POSSIBLY TAP HIDDEN GRAVITATIONAL/ELECTROMAGNETIC ENERGY FOR TECHNICAL PURPOSES

A relatedness of the gravitational and the magnetic energy problem is revealed by the following thought experiment: Given our two permanently magnetized disks (that attract each other) sit in a closed box, we find that, due to energy being proportional to mass, their weight in a gravity field is higher when they are side-by-side, and is lower when they are at a some distance from each other. (When the disks are sitting side by side, both the energy stored in the resulting magnetic field and also the energy stored in a compressed spring—sitting in the elevator cabin and receiving the mechanical work given off when the two disks are approaching each other—is increased.) Hence, when the box is in an elevator cabin, one can make the cabin's weight lower during the rise of the cabin by arranging that the two disks are some distance away from each other, and can make the cabin's weight higher during the cabin's descent by arranging that the two disks are side-by-side. One could thereby extract network in a cyclical process of raising and lowering the elevator cabin.

This suggests: The reservoir feeding flows of electromagnetic energy and the reservoir feeding flows of gravitational could be one and the same reservoir.

The thought experiment also reveals that there is, in principle, no obstruction to cyclic processes that might harness the hidden energy reservoir for technical purposes.

VIII. RESULTS

The following results have been established:

- The Poynting vector reveals that spinning electrons—which are mainly responsible for magnetism in permanent magnets—are the source of a flow of electromagnetic energy when two permanent magnets give in to their mutual attraction and are thereby yielding mechanical work.

- Those spinning electrons are subject to a relativistic electric field and to a Lorentz force that both try to obstruct the spinning of the electron, but are unsuccessful in achieving their goal.
- The energy delivered to the ambient—both in the form of an increase in the energy of the resulting magnetic field and in the form of mechanical work—during the mutual approach of two permanent magnets is equal in amount to the mechanical work needed to overcome the obstructive forces that (unsuccessfully) try to stop the spinning.
- As the electron does not house an energy reservoir in its interior that could provide the energy needed to maintain the spinning of the electron, the principle of local conservation of energy requires the existence of a nearby energy reservoir in a fourth spatial dimension. This is a (so-far undetected) consequence of the unstoppability of any electron spin.
- Thereby the dilemma presented by Mach's principle can be solved: The water in a spinning bucket climbs up the inner walls of the bucket, not—as has been believed—because the water rotates with respect to distant stars and galaxies (which would constitute a forbidden "action at a distance"), but because it rotates with respect to the inert mass of the nearby energy reservoir.

[1] R. Becker, *Electromagnetic Fields and Interactions*, Vol. I (Dover, New York, 1982).
[2] J. C. Maxwell, *Treatise on Electricity and Magnetism*, Vol. II (Dover, New York, 1954).
[3] O. Heaviside, *The Electrician*, Vol. 31 (1893), p. 281; also published in O. Heaviside, *Electromagnetic Theory*, Vol. 1 ('The Electrician' Printing and Publishing Company, London, 1898), p. 461.
[4] G. Mie, *Die Einsteinsche Gravitationstheorie* (Einstein's theory of gravitation) (Hirzel, Leipzig, 1921).
[5] G. Mie, *Lehrbuch Der Elektrizität Und Des Magnetismus (Textbook of Electricity and Magnetism)*, 3rd ed. (F. Enke, Stuttgart, 1948).
[6] A. Trupp, Phys. Essays **32**, 484 (2019).
[7] A. Trupp, Adv. Stud. Theor. Phys. **16**, 239 (2022).
[8] C. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Princeton University Press, Princeton, 1973).
[9] A. Trupp, J. Mod. Phys. **13**, 16 (2022).
[10] A. Sommerfeld, *Elektrodynamik (Electrodynamics)*, 5th ed. (Akademische Verlagsgesellschaft, Leipzig, 1967).
[11] E. M. Purcell, *Electricity and Magnetism*, 2nd ed., Vol. 2 (Berkeley Physics Course, McGraw Hill, New Delhi, India, 1985).
[12] A. Föppl, *Einführung in die Maxwellsche Theorie der Elektrizität (Introduction to Maxwell's Theory of Electricity)*, 3rd ed. (B. G. Teubner, Leipzig, 1907).
[13] M. Planck, *Einführung in die Theoretische Physik, Vol. III: Einführung in die Theorie der Elektrizität und des Magnetismus (Introduction to Theoretical Physics, Vol. III: Introduction to the Theory of Electricity and Magnetism)*, 2nd ed. (Hirzel, Leipzig, 1928).
[14] H. Reichenbach, *The Philosophy of Space and Time* (Dover, New York, 1957).
[15] K. Tuttle, Symmetry Mag. **2**, 18 (2005).
[16] L. Randall, *Warped Passages – Unraveling the Mysteries of the Universe's Hidden Dimensions* (Ecco, New York, 2005).
[17] L. Susskind and G. Hrabovsky, *The Theoretical Minimum* (Penguin, Toronto, 2013).
[18] A. Einstein, *Relativity: The Special and the General Theory* (Bonanza Books, New York, 1961).
[19] M. Abraham, Scientia **15**, 8 (1914).
[20] W. Wien, Ann. Phys. Chem. **281**, 685 (1892).
[21] A. Einstein, "The Foundation of the General Theory of Relativity" A. Einstein, H. Weyl, and H. Minkowski, *The Principle of Relativity, a Collection of Original Papers on the Special and the General Theory of Relativity* (Dover, New York, 1952).
[22] T. Rothman, Am. Sci. **105**, 344 (2017).